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Solution of the porous media equation by Adomian's decomposition method

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Abstract

The particular exact solutions of the porous media equation that usually occurs in nonlinear problems of heat and mass transfer, and in biological systems are obtained using Adomian's decomposition method. Also, numerical comparison of particular solutions in the decomposition method indicate that there is a very good agreement between the numerical solutions and particular exact solutions in terms of efficiency and accuracy.

MSC: 35Q53; 35B45

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1. Introduction

In this Letter, we consider the nonlinear heat equation called the porous media equation

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(u^m \frac{\partial u}{\partial x}\right), \tag{1}$$

where m is a rational number.

Finding the particular exact solutions that have a physical or biological interpretation for the nonlinear equations of the form (1) is of fundamental importance. This equation often occurs in nonlinear problems of heat and mass transfer, combustion theory, and flows in porous media. For example, it describes unsteady heat transfer in a quiescent medium with the heat diffusivity being a power-law function of temperature [12].

Eq. (1) has also applications to many physical systems including the fluid dynamics of thin films [11]. Murray [6] describes how this model has been used to represent "population pressure" in biological systems. Eq. (1) is called a degenerate parabolic differential equation because the diffusion coefficient $D(u) = u^m$ does not satisfy the condition for classical diffusion equations, $D(u) > 0$ [11]. For the motion of thin vis-

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Table 1
Numerical results for $|u(x,t) - \phi_{50}(x,t)|$ where $u(x,t) = \frac{1}{x-t}$ for Eq. (12)

$t_i \mid x_i$	0.5	0.6	0.65	0.7	0.8
0.11	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
0.2	0.0000E+00	1.3322E−15	1.3322E−15	0.0000E+00	0.0000E+00
0.3	4.0413E−11	2.6645E−15	0.0000E+00	0.0000E+00	0.0000E+00
0.4	1.4272E−4	7.8416E−9	1.1465E−10	2.3510E−12	2.2204E−15
0.45	0.1030E+00	3.7754E−6	5.1752E−8	1.0180E−9	9.1571E−13

cous films, (1) with $m = 3$ can be derived from the Navier–Stokes equations. Lacking a physical law to describe the complex behavior in a system, an appropriate value for the parameter m can be determined by comparing known solutions with empirical data [11].

In the next section, the Adomian's decomposition method (ADM) [1] is applied to Eq. (1) to obtain the particular exact solutions of it. It is well known that this method avoids linearization and physically unrealistic assumptions, and provides an efficient numerical solution with high accuracy [3,4,9,10].

2. The method

Eq. (1) can be written in an operator form

$$L_t(u) = L_x\big(u^m L_x u\big), \tag{2}$$

with the initial and boundary conditions, where the notations $L_t = \frac{\partial}{\partial t}$ and $L_x = \frac{\partial}{\partial x}$ symbolize the linear differential operators. We assume the integration inverse operators L_t^{-1} and L_x^{-1} exist, and they are defined as $L_t^{-1} = \int_0^t (\cdot)\,dt$ and $L_x^{-1} = \int_0^x (\cdot)\,dx$, respectively. Therefore, one can write the solution in t direction as [1]

$$u(x,t) = u(x,0) + L_t^{-1}\big[L_x\big(\Phi(u)\big)\big], \tag{3}$$

where $\Phi(u) = u^m u_x$. By ADM [1] one can write the solution in series form as

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t). \tag{4}$$

To find the solutions in t direction, one solves the recursive relations

$$u_0 = u(x,0), \qquad u_{n+1} = L_t^{-1}\big[L_x(A_n)\big], \quad n \geqslant 0, \tag{5}$$

respectively, where the Adomian polynomials are [1, 3,4]

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[\Phi\left(\sum_{n=0}^{\infty}\lambda^n u_n\right)\right]_{\lambda=0}, \quad n \geqslant 0. \tag{6}$$

We obtain the first few Adomian polynomials for $\Phi(u) = u^m u_x$ as

$$\begin{aligned}
A_0 &= u_0^m (u_0)_x,\\
A_1 &= m u_0^{m-1} u_1 (u_0)_x + u_0^m (u_1)_x,\\
A_2 &= m u_0^{m-1} u_2 (u_0)_x + m u_0^{m-1} u_1 (u_1)_x\\
&\quad + u_0^m (u_2)_x + \frac{m}{2}(m-1) u_0^{m-2} u_1^2 (u_0)_x,\\
A_3 &= m u_0^{m-1} u_3 (u_0)_x + m(m-1) u_0^{m-2} u_1 u_2 (u_0)_x\\
&\quad + \frac{m}{2}(m-1) u_0^{m-2} u_1^3 (u_0)_x + m u_0^{m-1} u_2 (u_1)_x\\
&\quad + m u_0^{m-1} u_1 (u_2)_x + u_0^m (u_3)_x,\\
&\vdots
\end{aligned}$$

The convergence of the decomposition series (4) is studied in [2].

In the following section we provide some examples and demonstrate the absolute errors $|u(x,t) - \phi_n(x,t)|$ in Tables 1–2, where $u(x,t)$ is the particular exact solution and $\phi_n(x,t)$ is the partial sum

$$\phi_n(x,t) = \sum_{k=0}^{n} u_k(x,t), \quad n \geqslant 0. \tag{7}$$

As it is clear from (4) and (7)

$$u(x,t) = \lim_{n\to\infty} \phi_n(x,t). \tag{8}$$

Equations of the form (1) admit traveling-wave solutions $u = u(kx + \lambda t)$ where k and λ are constants [12].