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Intermediate glassy phase for the mean-field Potts glass model in a field

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Abstract

The infinite-range p-state Potts glass model is studied in the presence of a field. Stability of the replica symmetric solution for the longitudinal order parameter is investigated. Instability appears at higher temperature than the transverse freezing temperature for $p > 3.2$. Replica symmetry breaking pattern is obtained.

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Keywords: Potts glass; Field; Replica symmetry breaking

1. Introduction

The p-state Potts glass model has been studied for a long time. After the introduction of the mean-field theory of the model [1,2], self-consistent description of the glass phase within the Parisi's replica theory was established [3,4]. The nature of the glass phase is rather different from the one for the mean-field theory of the Ising spin glass model. It is expected to be related to some features of the structural glass [5].

Although a short description on the Potts glass model in a field can be found in Ref. [6], most of the previous works concentrated on the zero field case.

In this Letter, the mean-field Potts glass model in a field is studied focusing on the stability of the replica symmetric solution above the transverse freezing temperature. Replica symmetry is shown to be broken at higher temperature than the transverse glass transition temperature for $p > 3.2$. The longitudinal order parameter function in this phase is obtained.

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2. The mean-field p-state Potts glass model

Using the simplex representation [7], the Hamiltonian of the mean-field p-state Potts glass model is given by

$$H = -\sum_{\langle ij\rangle}\sum_{a=1}^{p-1} J_{ij} S_{ia} S_{ja} - \sum_{i}\sum_{a=1}^{p-1} h_a S_{ia}, \tag{1}$$

where the sum of the first term is over all distinct pairs. The quenched random bond J_{ij} has a mean value J_0 and variance J^2/N with N number of sites. The uniform field h_a is applied. The spin $\mathbf{S}_i$ can be one of the $(p-1)$-dimensional vectors $\{\mathbf{e}^\lambda\}$, $\lambda = 1, \ldots, p$. These vectors satisfy the following relations:

$$\sum_{a=1}^{p-1} e_a^\lambda e_a^{\lambda'} = p\delta_{\lambda\lambda'} - 1, \tag{2}$$

$$\sum_{\lambda=1}^{p} e_a^\lambda e_b^\lambda = p\delta_{ab}, \tag{3}$$

and

$$\sum_{\lambda=1}^{p} e_a^\lambda = 0. \tag{4}$$

In the following, $a = 1$ is taken to be the direction of the uniform field.

Although the Potts model is not symmetric under the exchange of $h_a \leftrightarrow -h_a$, $h_a > 0$ is assumed here. This is because appearance of the intermediate glassy phase is seen more clearly for $h_a > 0$ as described below.

Using the replica method to take the random average for J_{ij}, the free energy per site f is given by the maximum condition

$$-\beta f = \lim_{n\to 0} \frac{1}{n} \max(A), \tag{5}$$

where $\beta = \frac{1}{k_B T}$ with k_B Boltzmann's constant, T the temperature, and n denoting number of replicas. A is given by

$$\begin{aligned} A = {} & \frac{\mu}{4}(p-1)n - \frac{\mu}{4}\sum_{\alpha\neq\beta}\sum_{a,b}\left(q_{ab}^{\alpha\beta}\right)^2 + \frac{1}{2}\beta\hat{J}\sum_{a,\alpha}\left(m_a^\alpha\right)^2 \\ & + \ln \mathrm{Tr}_{S_a^\alpha=\{e_a^\lambda\}} \exp\left[\frac{\mu}{2}\sum_{\alpha\neq\beta}\sum_{a,b} S_a^\alpha S_b^\beta q_{ab}^{\alpha\beta} + \beta\hat{J}\sum_{a,\alpha} S_a^\alpha m_a^\alpha + \beta\sum_{\alpha} S_a^\alpha h_a\right]. \end{aligned} \tag{6}$$

Here $\mu \equiv \beta^2 J^2$ and $\beta\hat{J} \equiv \beta J_0 + \frac{\mu}{2}(p-2)$. Glass order parameter $q_{ab}^{\alpha\beta}$ and ferromagnetic order parameter m_a^α are introduced. It should be noted that effective ferromagnetic interaction $\hat{J}$ grows as temperature decreases [1]. Expanding the exponential keeping terms to forth order in the order parameters, the trace can be evaluated. After re-exponentiation, A is written as

$$\begin{aligned} A \simeq {} & \frac{\mu}{4}(p-1)n - \frac{\mu}{4}\sum_{\alpha\neq\beta}\sum_{a,b}\left(q_{ab}^{\alpha\beta}\right)^2 + \frac{1}{2}\beta\hat{J}\sum_{a,\alpha}\left(m_a^\alpha\right)^2 \\ & + \frac{\beta^2}{2}\sum_{\alpha}\sum_{a}\left(\hat{J}z_a^\alpha\right)^2 + \frac{\mu^2}{4}\sum_{\alpha\neq\beta}\sum_{a,b}\left(q_{ab}^{\alpha\beta}\right)^2 + \frac{\beta^3}{6}\sum_{\alpha}\hat{J}z_a^\alpha\hat{J}z_b^\alpha\hat{J}z_c^\alpha\frac{v_{abc}}{p} + \frac{\beta^2\mu}{2}\sum_{\alpha\neq\beta}\hat{J}z_a^\alpha\hat{J}z_b^\alpha q_{ab}^{\alpha\beta} \end{aligned}$$