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Extended F-expansion method and periodic wave solutions for the generalized Zakharov equations

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Abstract

We present an extended F-expansion method for finding periodic wave solutions of nonlinear evolution equations in mathematical physics, which can be thought of as a concentration of extended Jacobi elliptic function expansion method proposed more recently. By using the extended F-expansion, without calculating Jacobi elliptic functions, we obtain simultaneously a number of periodic wave solutions expressed by various Jacobi elliptic functions for the generalized Zakharov equations. In the limit cases, the solitary wave solutions are obtained as well.
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1. Introduction

In this Letter we consider the generalized Zakharov equations (GZEs) for the complex envelope $\psi(x,t)$ of the high-frequency wave and the real low-frequency field $v(x,t)$ in the form [1,2]

$$i\psi_t + \psi_{xx} - 2\lambda|\psi|^2\psi + 2\psi v = 0, \tag{1}$$

$$v_{tt} - v_{xx} + \left(|\psi|^2\right)_{xx} = 0, \tag{2}$$

where the cubic term in Eq. (1) describes the nonlinear self-interaction in the high-frequency subsystem, such a term corresponds to a self-focusing effect in plasma physics. The coefficient λ is a real constant that can be a positive or negative number. The sound velocity and the coupling constant in Eq. (2) have been normalized to unity

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for simplicity. GZEs are universal model of interaction between high- and low-frequency waves in one dimension. The collisions between solitary waves of GZEs were simulated in detail in [1], and internal oscillations of the solitary wave were analyzed based on the variation approach in [2].

Recently many solutions expressed by various Jacobi elliptic functions for a wide class of nonlinear evolution equations in mathematical physics have been obtained by Jacobi elliptic function expansion method [3–10] and F-expansion method [11–17] which can be thought of as a generalization of Jacobi elliptic function expansion since F here stands for everyone of Jacobi elliptic functions. In the present Letter we will extend the F-expansion, by which to find more types of periodic wave solutions expressed by various Jacobi elliptic functions of GZEs (1)–(2) than those obtained by F-expansion, without calculating various Jacobi elliptic functions. The extended F-expansion can be regard as a concentration of extended Jacobi elliptic function expansion [18,19] proposed more recently. The extended F-expansion will be elucidated in detail by taking Eqs. (1) and (2) as an illustrative example.

The Letter is organized as follows: in Section 2, a general form of traveling wave solutions, i.e., concentration formulas of solutions to GZEs are obtained; in Section 3, from concentration formulas of solutions, many types of Jacobi elliptic function solutions are produced simultaneously, and under the limit cases the solitary wave solutions are given as well; in Section 4, some comments for the method are made.

2. Derivation of concentration formulas

In this section the concentration formulas of the solutions to GZEs (1)–(2), which includes all sorts of traveling wave solutions, will be derived by using the extended F-expansion.

(1) Since $\psi(x,t)$ in Eqs. (1)–(2) is a complex function we assume that the traveling wave solutions of Eqs. (1)–(2) is of the form

$$\psi(x,t) = e^{i\eta}u(\xi), \tag{3$_1$}$$

$$v(x,t) = v(\xi), \qquad \eta = \alpha x + \beta t, \quad \xi = k(x - 2\alpha t), \tag{3$_2$}$$

where $u(\xi)$ and $v(\xi)$ are real functions, the constants α, β and k are to be determined. Substituting (3) into Eqs. (1)–(2) and canceling $e^{i\eta}$ yields an ordinary differential equations (ODEs) for $u(\xi)$ and $v(\xi)$

$$k^2u'' + 2uv - (\alpha^2 + \beta)u - 2\lambda u^3 = 0, \tag{4$_1$}$$

$$k^2(4\alpha^2 - 1)v'' + k^2(u^2)'' = 0. \tag{4$_2$}$$

In order to simplify ODEs (4) further, integrating Eq. $(4)_2$ once and taking integration constant to zero, and integrating it again yields

$$v(\xi) = \frac{u^2}{1 - 4\alpha^2} + C, \quad \text{if } \alpha^2 \neq \frac{1}{4}, \tag{5}$$

where C-integration constant.
Substituting expression (5) into Eq. $(4)_1$ we have an ODE for $u(\xi)$

$$k^2u'' + (2C - \alpha^2 - \beta)u + 2\left(\frac{1}{1 - 4\alpha^2} - \lambda\right)u^3 = 0. \tag{6}$$

Now the main goal is to solve the ODE (6).

(2) Considering the homogeneous balance between u'' and u^3 in ODE (6), we assume that $u(\xi)$ can be expressed by the extended F-expansion in the following form

$$u(\xi) = a_{-1}F^{-1}(\xi) + a_1F^1(\xi) + b_1G(\xi), \tag{7}$$

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