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Limitation of time-delay induced amplitude death

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Abstract

The present Letter provides the sufficient condition for avoiding amplitude death in two identical nonlinear systems coupled by a delay diffusive connection. This condition is that if a Jacobi matrix at a fixed point in an isolated system has an odd number of real positive eigenvalues, then the time-delay connection never induces amplitude death at the fixed point for any coupling parameters. Furthermore, these results are valid for globally-coupled systems and general types of coupling.

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1. Introduction

Coupled nonlinear oscillators generate several interesting phenomena in numerical simulations and real experiments. *Amplitude death* in coupled oscillators has received considerable attention [1–4]: individual oscillators stop to oscillate by diffusive connections. This phenomenon is caused by a coupling-induced stabilization of a fixed point in diffusive coupled oscillators.[1] It was said that amplitude death only occurs when the oscillators have the wide distribution of natural frequencies. Hence, amplitude death never occurs with coupled identical oscillators [2,3].

From a real point of view, it is obvious that there exists the influence of the finite speed of signal propagation in coupled oscillators. Several researchers have recently studied time-delay coupled oscillators [5–7]. Reddy, Sen, and Johnston discovered an important phenomenon: time-delay diffusive coupling induces amplitude death, even for

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[1] The stabilization is a necessary condition for amplitude death. It should be noted that amplitude death may not occur when stable oscillations (i.e., limit cycle, quasiperiodic motion, or chaotic oscillation) and stabilized fixed points coexist.

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coupled identical oscillators [8]. This discovery has created considerable interest in the field of nonlinear science [9], and the theoretical analysis for simple limit cycle oscillators has been completed in detail [10]. In addition, this phenomenon has been observed experimentally in electronic circuits [11] and thermo-optical oscillators [12]. It was reported that a time-delay induced stabilization occurs in coupled *discrete-time* systems [13], and the stability analysis of the oscillators coupled by a one-way ring time-delay scalar connection was investigated [14]. Atay presented an analysis of total and partial amplitude death in weakly nonlinear systems coupled by delayed connections [15], and showed distributed delays enlarge stability regions of amplitude death [16]. In the case of no delay, the death can occur in dynamical coupled oscillators [17] and in parametric-modulated coupled oscillators [18].

The majority of previous theoretical studies on time-delayed induced amplitude death focused on simplified two-dimensional limit cycle oscillators near *Hopf bifurcations*. However, there have been few attempts at extending the theoretical studies to real high-dimensional oscillators instead of the simplified oscillators. Furthermore, if one wants oscillatory behavior in real coupled oscillators, amplitude death should be avoided. Therefore, it would be useful to obtain the sufficient conditions under which death never occurs. This is because the condition can be used to design individual oscillators when the death is undesirable.

This Letter investigates two identical m-dimensional nonlinear systems with a time-delayed diffusive vector connection. The main purpose of this Letter is to clarify the following: (i) amplitude death never occurs for a nondelay connection; (ii) amplitude death never occurs at a class of fixed points; (iii) the statements (i) and (ii) are extended to globally coupled systems and no-diffusive connection.

The following notation will be used throughout this Letter: $\mathbb{R}^m$ represents the set of real m vectors, $\mathbb{R}^{p\times m}$ denotes the set of real $p\times m$ matrices, and $\mathbb{N}$ represents the set of positive integer numbers. The transpose of $\boldsymbol{A}$ is denoted by $\boldsymbol{A}^T$, and $\boldsymbol{I}_m$ represents the m-dimensional identity matrix.

2. Coupled systems

Consider two identical continuous-time subsystems $\Sigma_{\alpha,\beta}$

$$\Sigma_\alpha : \begin{cases} \dot{\boldsymbol{x}}_\alpha(t) = \boldsymbol{F}[\boldsymbol{x}_\alpha(t)] + \boldsymbol{B}\boldsymbol{u}_\alpha(t), \\ \boldsymbol{y}_\alpha(t) = \boldsymbol{C}\boldsymbol{x}_\alpha(t), \end{cases} \qquad \Sigma_\beta : \begin{cases} \dot{\boldsymbol{x}}_\beta(t) = \boldsymbol{F}[\boldsymbol{x}_\beta(t)] + \boldsymbol{B}\boldsymbol{u}_\beta(t), \\ \boldsymbol{y}_\beta(t) = \boldsymbol{C}\boldsymbol{x}_\beta(t), \end{cases}$$

where $\boldsymbol{x}_{\alpha,\beta}(t)\in\mathbb{R}^m$ are the system variables, and $\boldsymbol{u}_{\alpha,\beta}(t)\in\mathbb{R}^l$ and $\boldsymbol{y}_{\alpha,\beta}(t)\in\mathbb{R}^p$ are the coupling signals. l and p are the dimensions of the signals $\boldsymbol{u}_{\alpha,\beta}(t)$ and $\boldsymbol{y}_{\alpha,\beta}(t)$, respectively. $\boldsymbol{F}:\mathbb{R}^m\to\mathbb{R}^m$ denotes the differentiable nonlinear function, and has a fixed point $\boldsymbol{x}^*$; that is, $\boldsymbol{0}=\boldsymbol{F}[\boldsymbol{x}^*]$. $\boldsymbol{B}\in\mathbb{R}^{m\times l}$ and $\boldsymbol{C}\in\mathbb{R}^{p\times m}$ are the coupling matrices. As shown in Fig. 1, these subsystems are coupled by

$$\boldsymbol{u}_\alpha(t) = \boldsymbol{K}\bigl\{\boldsymbol{y}_\beta(t-\tau) - \boldsymbol{y}_\alpha(t)\bigr\}, \qquad \boldsymbol{u}_\beta(t) = \boldsymbol{K}\bigl\{\boldsymbol{y}_\alpha(t-\tau) - \boldsymbol{y}_\beta(t)\bigr\}. \tag{1}$$

$\boldsymbol{K}\in\mathbb{R}^{l\times p}$ corresponds to the coupling strength and $\tau\geqslant 0$ denotes the delay time. Each of the signals $\boldsymbol{u}_{\alpha,\beta}(t)$ includes the delayed signal $\boldsymbol{y}_{\beta,\alpha}(t-\tau)$. This is a diffusive type coupling with delay.

Fixed points can be shifted to the origin via a change of variables [20]. Therefore, without a loss of generality, it will be always assumed that the fixed point of subsystems $\Sigma_{\alpha,\beta}$ without coupling ($\boldsymbol{K}=\boldsymbol{0}$) is the origin: $\boldsymbol{0}=\boldsymbol{F}[\boldsymbol{0}]$.

Each isolated subsystem is assumed to have an unstable fixed point $\boldsymbol{0}$. The subsystems $\Sigma_{\alpha,\beta}$, which are coupled by (1), have the fixed point

$$[\,\boldsymbol{x}_\alpha^T \quad \boldsymbol{x}_\beta^T\,]^T = [\,\boldsymbol{0}^T \quad \boldsymbol{0}^T\,]^T. \tag{2}$$

Delayed coupling (1) does not change the location of fixed point (2). It should be noted that the dynamics of a coupled system is equivalent to that described in paper [3] when $\tau=0$, $\boldsymbol{B}\boldsymbol{K}=d\boldsymbol{I}_m$ and $\boldsymbol{C}=\boldsymbol{I}_m$. A coupling-induced stabilization of fixed point (2) is a necessary condition for amplitude death.