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Transverse vibrations of double-walled carbon nanotubes under compressive axial load

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Abstract

Based on the Bernoulli–Euler beam theory, a double-elastic beam model is developed for transverse vibrations of double-walled carbon nanotubes under compressive axial load, which includes the interaction of van der Waals force between the inner and outer tubes and the effect of compressive axial load. Explicit expressions are derived for natural frequencies and associated amplitude ratios of the inner to the outer tubes for the case of simply supported double-walled carbon nanotubes. The influences of compressive axial load on the properties of vibrations are discussed. It is shown that the effects of compressive axial load on the natural frequencies of double-walled carbon nanotubes are sensitive to the vibration modes and aspect ratios. The natural frequencies are dependent on the axial load and decrease with increasing the axial load. However, the associated amplitude ratios of the inner to the outer tubes of double-walled carbon nanotubes are independent of the axial load. In addition, the critical axial buckling stress and strain for simply supported double-walled carbon nanotubes are obtained.

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Keywords: Transverse vibration; Double-walled carbon nanotubes; Compressive axial load

1. Introduction

Carbon nanotubes discovered in 1991 [1] have received extensive attention and drawn a great deal of research from various branches of science [2–10]. Carbon nanotubes can be produced by an array of techniques, such as

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arc discharge, laser ablation, and chemical vapor deposition. Depending on the synthesis conditions, nanotubes can be single-walled or multi-walled. A single-walled carbon nanotube is a cylinder of graphene with a single layer of carbon atoms, and its diameter is on the order of 1 nm. Multi-walled carbon nanotubes are cylinders of graphene with multiple layers of carbon atoms along the tube thickness, and their diameters are much larger. The length of carbon nanotubes can be as large as 10^4–10^5 times the diameter [6]. Both experimental and theoretical studies showed that carbon nanotubes have exceptional mechanical and electronic properties such as high stiffness-to-weight and strength-to-weight ratios and enormous electrical and thermal conductivities [4,11]. Due to their remarkable mechanical, physical and chemical properties, carbon nanotubes may be used as structural elements in nanoscale devices or potential reinforcements in nanocomposite materials [12,13].

In nanoscale devices, large stresses can occur on account of thermal or lattice mismatch between different materials. Consequently, the reliability of many devices depends critically on the understanding of the responses of carbon nanotubes to mechanical loading. Since most potential applications of carbon nanotubes are heavily based on a thorough understanding of their mechanical behavior [14,15], the study of mechanical behavior of carbon nanotubes has become one topic of major concern [4,5,10,16,17]. As experiments at the nanoscale are extremely difficult, theoretical modeling for mechanical response of carbon nanotubes has been carried out [6,11]. These modeling approaches generally include atomistic modeling and continuum mechanics modeling. As atomistic modeling is very time consuming and computationally expensive for large-sized atomic systems, continuum mechanics models have been widely used to study mechanical behavior of carbon nanotubes [4,10,16–21]. Many elastic continuum models have been applied to the investigation of the vibrational behavior of carbon nanotubes [22–25]. Sohlberg et al. [26] theoretically analyzed the vibration of carbon nanotubes by simply regarding nanotubes as solid slender rods. Kahn et al. [27] studied quantized vibrational modes of nanotubes in the elastic continuum model. Yoon et al. [28] investigated the intertube vibration of multi-walled carbon nanotubes by a multiple-elastic beam model.

In recent years, a lot of research has been devoted to the application of carbon nanotubes as chemical and mechanical sensors [29–34]. The basic principle of sensing is based on the natural (resonant) frequency shift of a carbon nanotube resonator when it is subjected to a axial strain resulting from external axial loading. It is found that the natural frequency is sensitive to the applied axial load. In consequence, the effect of axial load on the property of transverse vibration of carbon tubes is of practical interest. In this Letter, a double-elastic beam model is developed for transverse vibrations of double-walled carbon nanotubes under compressive axial load on the basis of Bernoulli–Euler beam theory. Following this double-elastic beam model, explicit expressions are derived for natural frequencies and associated amplitude ratios of the inner to the outer tubes for the case of simply supported double-walled carbon nanotubes, and the influences of compressive axial load on them are discussed. In addition, the critical axial buckling stress and strain for simply supported double-walled carbon nanotubes are derived.

2. Double-elastic beam model

Using the Bernoulli–Euler beam theory, which is based upon the assumption that plane cross-sections of a beam remain plane during flexure and that the radius of curvature of a bent beam is large compared with the beam's depth, the general equation for transverse vibrations of an elastic beam under compressive axial load and distributed transverse pressure is expressed by [17,35,36]

$$p(x) + Fw'' = EIw^{\mathrm{IV}} + \rho A\ddot{w}, \tag{1}$$

where $p(x)$ is the distributed transverse pressure per unit axial length, F is the compressive axial load, w is the transverse displacement of the beam, I and A are the second moment of area and the cross-sectional area of the beam, and E and ρ are Young's modulus and the mass density. Thus, EI denotes the bending stiffness of the beam, and ρA represents the mass density per unit axial length. In addition, we define

$$w' = \frac{\partial w}{\partial x}, \qquad \dot{w} = \frac{\partial w}{\partial t}, \tag{2}$$