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Multiplayer quantum games with continuous-variable strategies

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Abstract

Based on the extended case of the Cournot's duopoly, we investigate a case of three firms in which the different entanglement parameters can vary arbitrarily. An analytical formula is presented and some interesting features are demonstrated from it. Furthermore, we find that the quantum entanglement can make an arbitrary number of players cooperate to the largest extent.

Keywords: Quantum game; Entanglement; Multiplayer; Continuous variable

1. Introduction

The theory of quantum information has become an increasingly important focus of study. Many other fields such as calculations, code communication and communicate capability, etc., have combined with it and resulting in a series of new findings being discovered. Many interesting and surprising properties have been uncovered since the concept of quantum information was applied to classical game theory. First, Meyer showed the power of quantum strategies by which the player can always beat his classical opponent [1]. Then, Eisert et al. showed how the power of entanglement can be used to eliminate the dilemma which exists in classical game [2]. After that, noisy quantum games [3,4], multiplayer quantum games [5,6], continuous-variable quantum games [7,8] and other interesting aspects [9–15] were studied.

The properties of two player quantum games have been discussed extensively, but the case of multiplayer quantum games has received less attention. Multiplayer games contains many properties which do not exist in two player games. In practice, many situations should be represented by multiplayer game. So, from both a theoretical and a practice standpoint, the study of multiplayer games is important. In this Letter, we investigate a multiplayer quantum game which is extended from the case of Cournot's duopoly [16].

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Cournot's duopoly model of a two player case has been discussed by Du et al. [7,8]. They showed that when the players' qubits are entangled in an EPR state the best profits for the two firms results. By utilizing the quantum structure where every pair of players are entangling with each other [6], we arrive at an analytic formula of the profits for the case of three firms. By illustrating the variety of the sum of these players' profits with the variation of the three different parameters which present the extent of entanglement, we find the conditions to attain the highest of the sum are not unique. Furthermore, when all of the players, presented by qubit, entangled pairwise to EPR state, they will obtain the highest profits while maintaining the symmetry of the game. This conclusion may be applicable to arbitrary number of participators.

2. The extension of the Cournot's duopoly

The Cournot's duopoly is a model of two firms monopolizing the market of a certain commodity. We extend this model by adding the number of new firms. Suppose there are n firms monopolizing the market of a certain commodity. Every firm may decide its own quantity of product, and all of their products are the same. For convenience, we denote the quantity of the jth firm as q_j and the total quantity as Q, so $Q = q_1 + \cdots + q_n$. Suppose the price is $P(Q)$, then we have

$$P(Q) = \begin{cases} a - Q & \text{for } Q \leqslant a, \\ 0 & \text{for } Q > a. \end{cases} \tag{1}$$

Taking the cost of each product as c with $c < a$. For convenience, let $k = a - c$. Then the profits can be represented as

$$u_j(q_1, \ldots, q_n) = q_j(k - Q), \tag{2}$$

where j means the jth firm. So the Nash equilibrium of the game is

$$q_1^* = \cdots = q_n^* = \frac{k}{n+1}. \tag{3}$$

The profits at the equilibrium are

$$u_1(q_1^*, \ldots, q_n^*) = \cdots = u_n(q_1^*, \ldots, q_n^*) = \frac{k^2}{(n+1)^2}. \tag{4}$$

However, this equilibrium is not the optimal solution. If we restrict the quantities at

$$q_1' = \cdots = q_n' = \frac{k}{2n}. \tag{5}$$

The profits of the firms will be

$$u_1\left(\frac{1}{2n}, \ldots, \frac{1}{2n}\right) = \cdots = u_n\left(\frac{1}{2n}, \ldots, \frac{1}{2n}\right) = \frac{k^2}{4n}. \tag{6}$$

This is a better solution of the game. Actually, it is the highest profit they can ever attain while maintaining the symmetry of the game. However, they will never escape the Nash equilibrium because of selfishness. So, in the extended Cournot's duopoly, a dilemma-like situation also exists.

3. Quantum form of the model with three firms

We use three single-mode electromagnetic fields. The quantum structure is shown in Fig. 1. The game starts from the vacuum state $|0\rangle_1 \otimes |0\rangle_2 \otimes |0\rangle_3$. First, the state passes through a unitary operator $\hat{J}$ which is known to all