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Asset prices and efficiency in a Krebs economy

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ABSTRACT

I study the asset pricing implications and the efficiency of a tractable dynamic stochastic general equilibrium model with heterogeneous agents and incomplete markets along the lines of Krebs (2003a). Contrary to previous applications of these types of models, I find that generically the distribution of idiosyncratic shocks affects the risk premia of aggregate shocks and that the equilibrium is constrained inefficient in the sense that a planner can Pareto improve the equilibrium outcome by assigning different portfolio choices to agents. The inefficiency is caused by a 'portfolio externality': the average portfolio of the economy affects the portfolio return of each agent. The constrained efficient outcome can be achieved through linear taxes and subsidies that I characterize in closed-form.

1. Introduction

Since the theoretical work of Bewley (1986) and the quantitative work of Huggett (1993), Aiyagari (1994), and Krusell and Smith (1998), heterogeneous-agent general equilibrium models with incomplete markets have been widely applied in economics. Two particular applications are asset pricing and welfare economics. The former literature is largely motivated by the inability of the representative-agent, consumption-based asset pricing model to explain various asset pricing puzzles, most notably the equity premium puzzle and the risk-free rate puzzle. The latter literature, for instance Dávila et al. (2012), is motivated by the theoretical result that the general equilibrium with incomplete markets (GEI) is generically constrained inefficient.[2]

Since heterogeneous-agent models are typically analytically intractable, few theoretical results are known about the asset pricing and welfare implications of incomplete market heterogeneous-agent models, apart from a few exceptions. By

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[2] A property holds "generically" if it holds for all parameter values except those in a set with measure zero.

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judiciously constructing individual income processes, Constantinides and Duffie (1996) show that any arbitrage-free asset prices, dividends, and aggregate consumption can be explained. Krueger and Lustig (2010), on the other hand, show that idiosyncratic labor income risk has no effect on the equity premium if idiosyncratic shocks are independent of aggregate shocks and aggregate consumption growth is independent over time. With regard to efficiency, Krebs (2003a, 2003b, 2006) develops a tractable dynamic general equilibrium model with heterogeneous agents who are subject to uninsurable idiosyncratic human capital risk[3] but finds that the equilibrium is nevertheless constrained efficient. This paper extends these two literatures by generalizing the model of Krebs (2006).

The contribution of this paper is twofold. First, I show that the irrelevance result of Krueger and Lustig does not generally hold unless there is a single source of aggregate risk. When there are multiple sources of aggregate risk, as in Constantinides and Duffie (1996), risk premia on assets are typically affected by idiosyncratic shocks. This is because when there are multiple sources of aggregate risk, the aggregate component of individual consumption growth (which prices assets through the Euler equation) depends on the portfolio choice, which in turn is affected by idiosyncratic shocks.

Second, I study the efficiency of the Krebs economy. Both in the baseline model of Krebs (2006) (with one technology, two inputs, and idiosyncratic human capital depreciation shocks) and the extension of Toda (2014) (with multiple *AK* technologies), the equilibrium is constrained efficient. However, in the more general case with one or more technologies/inputs and general idiosyncratic technological shocks, I find that the equilibrium is generically constrained inefficient. The inefficiency is cause by a pecuniary externality (Greenwald and Stiglitz, 1986): when there are multiple technologies and inputs, the return on an individual portfolio depends on other agents' portfolio (weighted average portfolio). Nevertheless, the constrained efficient outcome can be achieved through linear taxes and subsidies that can be characterized in closed-form. This result is in sharp contrast to those in the literature. For instance, the mathematical economics literature[4] typically studies only two period models, and while it is possible to show the existence of a Pareto improving intervention (Citanna et al., 1998), it is rarely possible to characterize the optimal intervention. Dávila et al. (2012) numerically solve a stochastic growth model with idiosyncratic labor income risk and find that the optimal tax rates depend on the wealth level and idiosyncratic states of agents. Being a quantitative work, however, they neither prove that the equilibrium is actually constrained inefficient nor that an optimal intervention exists.

I also provide a numerical example calibrated to the U.S. economy. The effect of idiosyncratic shocks on asset prices is substantial (the equity premium ranges from 0.63% to 3.74% by changing the idiosyncratic labor income volatility from 0% (representative-agent model) to 30%), and the welfare loss is moderate (1% in consumption equivalent). In this example (regardless of the parameter values) there is always an over-investment in capital, and the optimal tax rate on the capital stock is 0.5% in the baseline specification.

This paper is closest to Krebs (2003a, 2003b, 2006), Gottardi et al. (2011), and Toda (2014). Krebs (2006) is probably the first tractable dynamic general equilibrium with a continuum of heterogeneous agents and incomplete markets in a general (non-i.i.d.) Markov setting, which is particularly useful for various applications such as the current paper. His earlier works (Krebs, 2003a, 2003b) study the growth and welfare effects of human capital risk in specialized models. Gottardi et al. (2011) study the Ramsey problem of finding the optimal public debt and linear taxes in a model similar to the current paper (but with no aggregate risk and i.i.d. idiosyncratic risk). Although I do not consider public debt, my paper is complementary since I study a more general model and characterize the optimal tax rates (which happen to be linear) in closed-form. Toda (2014) studies the theoretical properties such as equilibrium existence and uniqueness with general preferences and shocks (which the current paper applies) and characterizes the stationary wealth distribution and the power law exponents.

2. A simple example

In this section I present a simple specialized model in order to build the intuition for the main results. The exposition is deliberately informal since formal definitions, theorems, and proofs in a more general model will be given in subsequent sections.

2.1. Settings

There is an "all purpose" good that can be either consumed or saved as physical and human capital.[5] Time is infinite and is denoted by $t = 0, 1, \ldots$. A perfectly competitive firm produces the good using the production function $F(K, H) = AK^{\alpha}H^{1-\alpha}$, where K, H are the efficiency units of physical and human capital, $A > 0$, and $0 < \alpha < 1$. Suppressing the time subscript, the firm's problem at each period is

[3] Similar models (*AK* models with idiosyncratic investment risk) have been used by Saito (1998), Angeletos (2007), and Toda (2014), among many others.

[4] The notion of constrained efficiency was first defined by Diamond (1967). Geanakoplos and Polemarchakis (1986) first proved the generic constrained inefficiency of equilibrium in a two period exchange economy. Geanakoplos et al. (1990) and Carvajal and Polemarchakis (2011) treat the case with production and idiosyncratic risk, respectively.

[5] If the reader is uncomfortable with the assumption that physical and human capital can be converted 1:1, one can interpret human capital as private equity.