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Stochastic description of the dynamics of a random-exchange Heisenberg chain

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Abstract

We study the diffusion process in a Heisenberg chain with correlated spatial disorder, with a power spectrum in the momentum space behaving as $k^{-\beta}$, using a stochastic description. It establishes a direct connection between the fluctuation in the spin-wave density of states and the noise density of states. For continuous ranges of the exponent β, we find superdiffusive and ballistic spin-wave motions. Both diffusion exponents predicted by the stochastic procedure agree with the ones calculated using the Hamiltonian dynamics.

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1. Introduction

In the last decades, a considerable number of dynamical systems have been studied and a great deal of attention has been paid to the analysis of their transport properties. In particular, the study of diffusion and transport properties of physical systems with short or long-range correlations in the disorder distribution has attracted a renewed interest [1–12]. For instance, the unexpected high conductance of several doped quasi-one-dimensional polymers was explained by assuming pairwise correlations in the disorder distribution [3]. Similarly, the suppression of Anderson localization was recently confirmed experimentally in semicon-

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ductor superlattices with correlated disorder [13]. Further, it was demonstrated that long-range correlations in site also act towards the delocalization of 1D quasiparticle states [10,11]. The 1D Anderson model with long-range correlated diagonal disorder displays a finite phase of extended states in the middle of the band of allowed energies, with two mobility edges separating localized and extended states [10]. This result was experimentally validated by microwave transmission spectra through a single-mode waveguide with inserted correlated scatterers [14]. The above results have motivated the study of other model systems that can be mapped onto the Anderson model, such as magnetic [15] and harmonic chains [16].

In the context of stochastic processes, Morgado et al. [1], studied diffusion in systems with long-range time correlation. First, they establish a direct connection between the noise density of states, $\rho_n(\omega)$, and the diffusive process. Second, they conjecture that the dynamics of a Hamiltonian system with space correlated disorder could be described by the same formalism if one supposes that the fluctuation in the density of states of the quasi-particle or elementary excitation, $\rho_F(\omega)$, plays the same role as ρ_n in the stochastic description. In this Letter, we present a numerical analysis of the validity of this conjecture. We study it for the one-dimensional quantum Heisenberg model exhibiting long-range correlations in the random exchange couplings. For continuous ranges of the degree of correlation, this system presents superdiffusive and ballistic motions [15]. Here, we provide numerical evidence that the Hamiltonian description and the stochastic one can be unified through the referred conjecture, thus confirming early expectations [1].

2. Stochastic and Hamiltonian descriptions

Let us suppose that the equation of motion for an operator A can be cast in the form [17–20]

$$\frac{dA(t)}{dt} = -\int_0^t \Gamma(t-t')A(t')\,dt' + h(t), \tag{1}$$

where $h(t)$ is a stochastic noise subject to the conditions $\langle h(t)\rangle = 0$, $\langle h(t)A(0)\rangle = 0$, and to the fluctuation–dissipation theorem [17]:

$$C_h(t) = \langle h(t)h(0)\rangle = \langle A^2\rangle \Gamma(t). \tag{2}$$

Here, $\langle\cdots\rangle$ indicates an ensemble average in thermal equilibrium. In principle, the presence of the kernel $\Gamma(t)$ allows us to study a large number of correlated processes. For example, in analogy with the usual Langevin's equation, we can study the asymptotic behavior of the second moment of the variable,

$$\sigma(t) = \int_0^t A(s)\,ds, \tag{3}$$

namely

$$\lim_{t\to\infty} \frac{\langle\sigma^2(t)\rangle}{t^\alpha} = K, \tag{4}$$

where K is a constant. In Eq. (4), we have $\alpha = 1$ for normal diffusion; for sub- and super-diffusion, $\alpha < 1$ and $\alpha > 1$, respectively.

The generalized field, $h(t)$, in Eq. (1) can be modeled by a thermal bath composed of harmonic oscillators; consequently, according to Eq. (2), the memory function can be written as

$$\Gamma(t) = \int \rho_n(\omega)\cos(\omega t)\,d\omega, \tag{5}$$

where $\rho_n(\omega)$ is the noise density of states. It has been proved [1] that *if the Laplace transform of the memory function of a unidimensional system behaves as*

$$\tilde{\Gamma}(z \to 0) \propto z^\nu, \tag{6}$$

then the diffusion exponent is

$$\alpha = \nu + 1. \tag{7}$$

In disordered Hamiltonian systems the diffusion process can be studied through direct integration of the equations of motion [15,16]. Now then, how can we assure that the two approaches are compatible and lead to the same results? For those systems the density of states of the quasi-particle or of the elementary excitation, $\mathbf{D}(E)$, plays the most significant role. However, it displays fluctuations, which are intrinsically connected to the diffusive behavior. Here, we conjecture that, for the relaxation processes, the fluctuation in the density of states, $\rho_F(E)$, plays the same role as the noise density of states in the stochastic process, thus

$$\rho_n(E) \leftrightarrow \rho_F(E). \tag{8}$$