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Mode-bifurcation upon pouring water into a cup that depends on the shape of the cup

Satoshi Nakata [a,*], Akiko Yamada [a], Hiroyuki Kitahata [a,b]

[a] *Department of Chemistry, Nara University of Education, Takabatake-cho, Nara 630-8528, Japan*
[b] *Department of Physics, Graduate School of Science, Kyoto University, Kyoto 606-8502, Japan*

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Abstract

Mode-bifurcation that included hysteresis was investigated when water was poured into a cup via a water faucet. Three modes, i.e., accumulation flow (mode I), scattering flow (mode II), and open and shut oscillatory flow (mode III), could be produced by laterally changing the distance between the center of the cup and the landing point of water poured at a constant flow rate. Hysteresis in mode-bifurcation was observed when the distance was changed depending on the initial location of the water poured into the cup. Pendulum flow was also observed when water was poured into a shallow cell. The essential features of this mode-switching were reproduced by a numerical calculation as a function of the landing point of the poured water using two-variable (the size of the water hollow and the pressure on the water surface) differential equations that included the nature of reversed flow.

1. Introduction

Many experimental systems with liquid flow or droplets have been used as simple and easily reproducible methods for studying various nonlinear phenomena such as oscillation, chaos, bifurcation, and pattern formation [1–16]. For example, the experiment with a dripping faucet introduced by Shaw has been used as an appropriate system for investigating chaotic phenomena [1–5]. Bohr and coworkers investigated the spatial nature of the hydraulic jump when a liquid flow hits onto a horizontal plate [6–9]. The “salt water oscillator”, i.e., periodic saline and water flow through an orifice in a cell [10–14], shows various types of synchronization and hysteresis. The collision of a droplet or flow with a liquid surface, which is famous as the fascinating “milk crown”, has been investigated experimentally and numerically [15,16].

Recently, we found that three modes of flow (accumulation flow (mode I), scattering flow (mode II),

* Corresponding author.
E-mail address: nakatas@nara-edu.ac.jp (S. Nakata).

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and concentric oscillatory flow (mode III)) were observed depending on the flow rate, Q, when water was poured from a water faucet into a cell [17,18]. A large hysteresis in mode-bifurcation was observed as a function of the flow rate and the radius of the tube. These three modes and hysteresis among modes I, II, and III were reproduced by a numerical calculation depending on the flow rate and the radius of the tube. In addition, we reported that the profile of the water hollow in mode-bifurcation was an important factor in reversed flow [18].

In this study, we examined the mode-bifurcation of water flow poured into a cell depending on the landing point of the water poured into the cell. Three modes, i.e., accumulation flow (mode I), scattering flow (mode II), and open and shut oscillatory flow (mode III), could be produced by laterally changing the location of the symmetric cell at a constant flow rate. Hysteresis in mode-bifurcation was observed depending on the scanning direction of the landing point of the poured water. Pendulum flow was also observed when water was poured into a shallow cell. The essential features of this mode-switching were qualitatively reproduced by a numerical calculation based on the two-variable differential equations that included the nature of reversed flow. We believe that this simple experimental system may be an appropriate model for clarifying mode-bifurcation and hysteresis.

2. Experiments

Fig. 1 shows a schematic representation of the experimental apparatus. A glass tube (length: 30 mm, inner radius: 1.25 mm) was connected to a water faucet via a flow meter (STEC Inc., LF10-PTN, Kyoto, Japan, minimum precision: 1.7 ml s^{-1}). Two symmetric ceramic cells ((cell α) height: 20 mm, radius at the top: 16.5 mm, radius at the bottom: 3.5 mm, volume: 8 ml, (cell β) height: 18 mm, radius at the top: 31.5 mm, radius at the bottom: 10 mm, volume: 40 ml) were used in this study. The ceramic cell was placed about 40 mm below the edge of the vertical tube. l $(= r_0 L)$ in Fig. 1 was defined as the horizontal distance between the center of the cell and the landing point of the poured water. The flow rate, Q, was carefully adjusted with the handle of the water faucet to unify the experimental conditions. Although

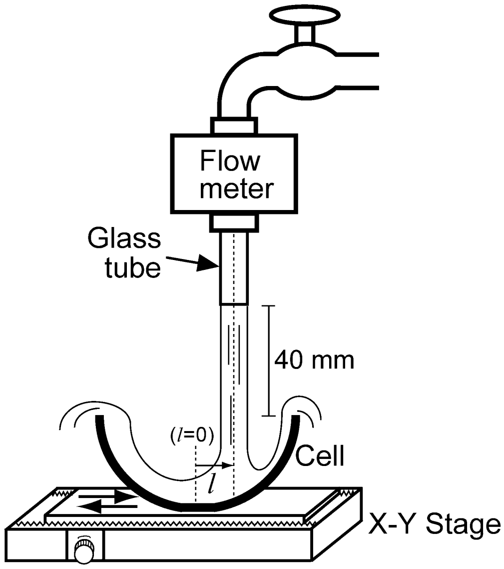

Fig. 1. Schematic representation of the experimental system. A cell was put on an x–y stage and the horizontal distance between the center of the cell and the landing point of the poured water, l, was changed using the x–y stage.

the use of this handle introduced a technical error in the change in the flow rate, the observed phenomena were sufficiently stable. The lateral location of the cell was changed with an x–y stage (minimum graduation of measure: 0.1 mm). All experiments were performed under room temperature. The experimental phenomena were monitored with a digital video camera (SONY, DCR-VX700), recorded on videotape, and then analyzed by a HIMAWARI system (LIBRARY Inc., Japan).

3. Results

Fig. 2 shows a bifurcation diagram among modes I, II, and III on the surface area of the water hollow (S_A) depending on l in cell α. The characteristic features of these modes are shown in snapshots. In mode I, the cell was filled with water. In mode II, water was scattered with the formation of a thin flow (a few millimeters thick) along the surface of the cell without filling the cell with water. In mode III, the magnitude of the water hollow, which corresponds to the region below water level that is not filled with water, oscillated periodically around the poured water. The os-