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Effects of orientation and symmetry of rods on the complete acoustic band gap in two-dimensional periodic solid/gas systems

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Abstract

We study numerically the acoustic band structures of five different shapes of steel rods (regular triangle, square, hexagon, octagon prisms and columns) placed, respectively, in air with a square lattice. The dependences of the complete acoustic band gaps (CABGs) on the orientation of the above noncircular rods and the maximum of CABG on the rods' symmetry are discussed.

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Keywords: Phononic crystal; Band gap; Symmetry

1. Introduction

During the past decade, the research on acoustic or elastic wave propagating in periodic composite materials, so-called phononic crystals, has received increasing attention because of their extensive potential applications as well as rich physics. One of the remarkable features of such artificial composites is their complete acoustic band gaps (CABGs) or phononic band gaps, in which wave propagation are prohibited [1–3]. As the mixed solid/gas combinations have the advantage of opening wide CABGs and, possess the necessary physical characteristics for practical application, phononic crystals consisting of an array of solid rods in air have been widely studied previously [4–17]. Theoretically, Caballero et al. [4] and Lai et al. [5] have studied the influence of lattice symmetry on CABGs in two-dimensional (2D) phononic crystals with rigid cylinders arranged in air. Papers [6] and [7] studied the rotation effect of hard square rods on CABGs. It is also reported that the CABGs change with the altering of

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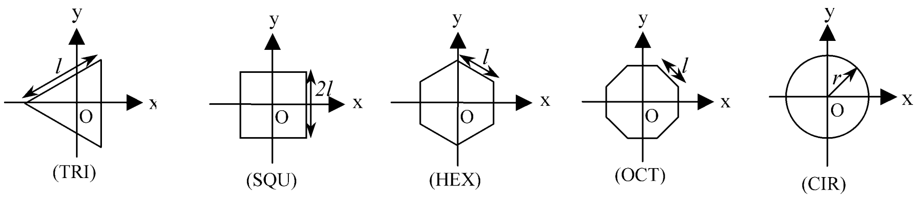

Fig. 1. Variously shaped rods and their dimensions. The cross-sections are regular triangle (TRI), square (SQU), hexagon (HEX), octagon (OCT) and circle (CIR), respectively. The axes passing through the center points O's are 3-, 4-, 6-, 8-, ∞-fold symmetry axes accordingly, hence, the rods' symmetry increases orderly.

microstructure for a square array of steel cylinders in air [8]. Meanwhile, some experimental measurements have been made, the results of which turn out to be in agreement with theoretical computations [4,9–13,17]. However, these studies involve only circular and square rods, and the role of the rods' symmetry played in the CABGs has not yet been analyzed systematically. In this Letter we studied the CABGs in 2D steel/air systems including five different shapes of rods, namely, regular triangle, square, hexagon, octagon prisms and columns (in order of symmetry increasing), and we find that the CABGs are related to the rods' orientation for those with noncircular ones. By adjusting the rods' orientation we can obtain the maximum width of CABGs at any filling fraction and reach some significant conclusions.

The approach we adopted to calculate in this Letter is the plane wave expansion (PWE) method [1,3,7,16,18–22]. Theoretically, PWE method fails to predict accurately the acoustic band structures for such a mixed solid/gas system. But due to the high density contrast between steel and air ($\rho_{\mathrm{steel}}/\rho_{\mathrm{air}} \approx 6116$), the penetration of sound inside the solid and, then, the effect of transversal waves can be fully disregarded [6,7,11,12,15]. Under this approximate treatment the solid is effectively treated as fluid. This assumption and the approximate PWE band structures have proven to agree well with those computed by finite difference time domain (FDTD) method [12] and the transfer matrix (TM) method [14].

2. Models

The critical part of computing with PWE method is the calculation of the rods' structure factors $P(\vec{G})$, which is absolutely determined by the shape of the rods [1,3,7,14–16,20–22]. In the present Letter we examined straight rods of five different cross-sections, i.e., regular triangle, square, hexagon, octagon and circle, whose symmetry increases orderly. Fig. 1 shows their cross-sections and dimensions. Among the five kinds of rods at the orientation as defined in Fig. 1, the structure factors for square [18,20,21], hexagon [21,22] and circle [22] cross-sectioned rods have been given in previous works, and the other two can be expressed as follows.

2.1. Triangle (TRI)

$$\begin{cases} \operatorname{Re} P^{\mathrm{tri}}_{G_x,0} = \frac{S^{-1}}{3G_x^2}\left[3lG_x \sin\left(\frac{\sqrt{3}lG_x}{6}\right) + 2\sqrt{3}\cos\left(\frac{\sqrt{3}lG_x}{6}\right) - 2\sqrt{3}\cos\left(\frac{\sqrt{3}lG_x}{3}\right)\right], \\ \operatorname{Im} P^{\mathrm{tri}}_{G_x,0} = \frac{S^{-1}}{3G_x^2}\left[3lG_x \cos\left(\frac{\sqrt{3}lG_x}{6}\right) - 2\sqrt{3}\sin\left(\frac{\sqrt{3}lG_x}{6}\right) - 2\sqrt{3}\sin\left(\frac{\sqrt{3}lG_x}{3}\right)\right], \end{cases} \tag{1a}$$

$$\begin{cases} \operatorname{Re} P^{\mathrm{tri}}_{G_x,\pm\sqrt{3}G_x} = \frac{S^{-1}}{6G_x^2}\left[\sqrt{3}\cos\left(\frac{\sqrt{3}lG_x}{3}\right) - \sqrt{3}\cos\left(\frac{2\sqrt{3}lG_x}{3}\right) + 3lG_x \sin\left(\frac{\sqrt{3}lG_x}{3}\right)\right], \\ \operatorname{Im} P^{\mathrm{tri}}_{G_x,\pm\sqrt{3}G_x} = \frac{S^{-1}}{6G_x^2}\left[\sqrt{3}\sin\left(\frac{\sqrt{3}lG_x}{3}\right) + \sqrt{3}\sin\left(\frac{2\sqrt{3}lG_x}{3}\right) - 3lG_x \cos\left(\frac{\sqrt{3}lG_x}{3}\right)\right], \end{cases} \tag{1b}$$