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The nonextensive gas: a kinetic approach

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Abstract

We discuss a kinetic nonextensive generalization of the Maxwellian ideal gas. The analysis rests on two basic assumptions: (i) instead of the standard Gaussian form, the q-gas is described by a power-law velocity distribution as suggested in the nonextensive Tsallis' framework (ii) the q-nonextensive generalization of the Boltzmann entropy formula governs the behavior of the q-gas. In this context, we show that the pressure and the internal energy are kinetically modified, but the general equation of state, $PV = 2U/3$, remains valid. The adiabatic index is now a function of the nonextensive parameter, $\gamma = C_p/C_V = 5/3q$. However, the standard expression relating the specific heats (at constant pressure and volume) with the coefficient of expansion and the isothermal compressibility, $C_P - C_V = TV\alpha^2/\kappa_T$, is not modified.
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It is widely known that all gases experience a strong tendency to have a universal behavior at the limit of very high temperatures and/or very low densities. The so-called perfect or ideal gas is an idealization of this limiting situation, being formally defined as a system of N free noninteracting particles. In the kinetic approach, the collisional equilibrium of a classical dilute gas is usually described by the celebrated Maxwellian velocity distribution which is a stationary solution of the Boltzmann transport equation. Its entropy is also a very well defined quantity which can be calculated through the standard Boltzmann expression [1,2]

$$-\frac{S}{k_B V} = \int f \ln f \, d^3 v. \tag{1}$$

The function $f(v)$ is the Maxwellian distribution

$$f(v) = Ae^{-mv^2/k_B T}, \tag{2}$$

where $A = n(m/2\pi k_B T)^{3/2}$ is the dimensional normalization constant ($n = N/V$ is the concentration, T

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is the temperature, and k_B is the Boltzmann constant). By inserting the above expression into (1) and carrying out the integral one finds that the entropy takes the form

$$-\frac{S}{k_B V} = n\left(\ln\left[n\left(\frac{m}{2\pi k_B T}\right)\right] - \frac{3}{2}\right). \tag{3}$$

In addition, if one assumes that the parameters T and V are the independent thermodynamic variables, or equivalently, $S = S(T, V)$, the differential expression corresponding to the above entropy formula reads

$$dS = Nk_B\left(\frac{dV}{V} + \frac{3}{2}\frac{dT}{T}\right). \tag{4}$$

The remaining basic thermodynamic relations (pressure and internal energy) for an ideal gas follows from the above expression by comparing it with the so-called Gibbs law ($T\,dS = dU + p\,dV$). As widely known, the specific heats, and relations like $PV = 2U/3$, as well as other important measurable quantities are immediately computed if one introduces the kinetic definition of the pressure [1]

$$P = \frac{m}{3}\int f v^2\, d^3 v. \tag{5}$$

On the other hand, the kinetic foundations of the nonextensive formalism [3–5] has been extensively discussed in the literature [6,7]. Instead of the standard Gaussian expression adopted in the extensive kinetic theory, the new approach is based on a power-law distribution. In this case, the distribution function reads (see, for instance, [7]):

$$f(v) = B\left(1 - (1-q)\frac{mv^2}{2k_B T}\right)^{1/1-q}, \tag{6}$$

where the constant B is also defined by the normalization condition

$$\int f(v)\, d^3 v = n, \tag{7}$$

being explicitly a q-dependent quantity, namely,

$$B = \begin{cases} A(1-q)^{1/2}\frac{5-3q}{2}\frac{3-q}{2}\Gamma\left(\frac{1}{2} + \frac{1}{1-q}\right)/\Gamma\left(\frac{1}{1-q}\right) \\ \quad \text{if } 1/3 < q \leqslant 1, \\ A(q-1)^{3/2}\Gamma\left(\frac{1}{q-1}\right)/\Gamma\left(\frac{1}{q-1} - \frac{3}{2}\right) \\ \quad \text{if } q \geqslant 1. \end{cases} \tag{8}$$

As expected, in the limiting case $q = 1$, both expressions reduce to the standard Maxwellian ones [7].

Recently, some authors have discussed the ideal gas within the nonextensive scenario working in the ensemble theory [8–13]. Here we will explore a different route. The main properties of the ideal nonextensive gas will discussed in terms of a kinetic formulation. The basic aim is to determine the analytic expressions generalizing the standard kinetic equilibrium approach describing the extensive ideal gas. As we shall see, the whole argument is quite similar to what happens in the extensive treatment. For instance, in order to compute the differential dS_q, we consider the new expression for the entropy, and the associated nonextensive distribution. This result it will be compared with the standard thermodynamic expression for the Gibbs law which must be obeyed by any gaseous system. As we shall see, this approach allow us to obtain the internal energy, the pressure, and the equation of state. In addition, by computing the specific heats (C_p and C_v), it will be seen that the standard expression relating such quantities with the response functions (isothermal compressibility and the coefficient of expansion) is not modified.

The kinetic version of Tsallis' entropy for an homogeneous and isotropic gas occupying a volume V can be written as [7]

$$-\frac{S_q}{k_B V} = \int B_0\left(\frac{f}{B_0}\right)^q \ln_q\left(\frac{f}{B_0}\right) d^3 v, \tag{9}$$

where the dimensional constant B_0 has been introduced by mathematical convenience in order to make the argument of the function $\ln_q f$ dimensionless. Note that if $q = 1$ the standard relation is recovered up to an irrelevant integral factor. The q-logarithmic function is defined as [4]

$$\ln_q f = \frac{f^{1-q} - 1}{1-q}, \quad f, q > 0. \tag{10}$$

The basic property of S_q is the non-additivity for $q \neq 1$. Given two independent systems A and B, the entropy composition rule verifies [3]

$$S_q(A+B) = S_q(A) + S_q(B) + (1-q)k_B^{-1} S_q(A) S_q(B).$$

In what follows, we focus our attention in the kinetic description of the particles using the power law distribution function given by (6). To begin with, we recall that the existence of statistical correlations are

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