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Entanglement change of mixed states under canonical unitary operations in two qubits

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Abstract

We investigate entanglement change of mixed states in a class by applying canonical unitary form of two-qubit unitary operations. The class, including Werner states, Bell diagonal states and maximum entangled mixed states, is characterized by three real parameters and geometrically presented by a tetrahedron. We propose a class of mixed states whose entanglement cannot be increased by any canonical unitary operation.

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Keywords: Entanglement; Canonical unitaries; Maximally entangled mixed states

1. Introduction

Quantum states, as well as quantum operations including unitary operations and measurements, are a useful physical resource and play an essential role in quantum information processing (QIP) [1]. The former are the carriers of information and used to encode quantum information, while the latter as a quantum dynamical resource manipulate the latter. It is of significant importance to study their properties not only for experiment but for pure theory. The properties for the latter can be studied via those of the former, and vice versa. The properties with respect to quantum unitary operations, such as entangling capability or power [2,3], classification and equivalence classes [4,5], have been studied via quantum states. On the other hand, some properties of quantum states under quantum operations was found, the bound entangled states is the case [6]. In particular, Ishizaka and Hiroshima [7] investigated the entanglement of two-qubit states operated by unitary operations, and found maximally entangled

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mixed states (MEMS's). Those states have an interesting property that their entanglement cannot be increased by any unitary. Further study showed MEMS's are the only ones with this property [8]. As well known, any two-qubit unitary can be decomposed into local ones and completely nonlocal one [1,9]. The completely nonlocal one is called canonical unitary, which are widely used in QIP [1,2,5,9]. Generally, the process of a unitary operation on a state is equivalent to three steps: first, local unitary operates on a state, and then canonical is applied, finally another local one is applied. However, in many situations one has the freedom to select the canonical unitary, while the local unitaries remain fixed. Indeed, this Letter was motivated by the study of interacting spin chains [10], where the unitary operation is the time evolution under a nearest-neighbour Hamiltonian, and is therefore subject to restrictions of the kind just mentioned above. In this Letter, we study in detail the entanglement change of a class of two-qubit mixed states by canonical unitary operations. The considered class of mixed states contains some important quantum states such as the MEMS's, Werner states [11] and Bell diagonal states [12], and is characterized by three parameters and geometrically presented by a tetrahedron. By calculating the entanglement changes of the states in the tetrahedron, we find a class of states with the property that their entanglement cannot be increased by any canonical unitary operation. We call the class of states maximally entangled mixed states under canonical unitary operation (MEMSCUO for short). MEMSCUO class contains MEMS class because of our restricted unitary. A possible application of MEMSCUO in QIP and comparison between MEMSCUO and MEMS are also given.

The Letter is organized as follows, in Section 2, the decompositions of two-qubit mixed state and unitary operation are reviewed, in Section 3, we investigate the entanglement change in detail, and find the class of MEMSCUO. In addition, the comparison of MEMSCUO with MEMS, and possible application are also given in the section. The Letter ends in Section 4 with a brief conclusion.

2. Two-qubit mixed states and decomposition of unitary operations

We start with a brief review on the decompositions of two-qubit quantum state and unitary operation. An arbitrary mixed two-qubit state can be written in terms of Pauli operators as

$$\mathrm{M}_{AB} = \frac{1}{4}\left(\mathbb{I}_2 \otimes \mathbb{I}_2 + \vec{u}\cdot\vec{\sigma}\otimes\mathbb{I}_2 + \mathbb{I}_2\otimes\vec{v}\cdot\vec{\sigma} + \sum_{i,j=1}^{3}\beta_{i,j}\sigma_i\otimes\sigma_j\right), \tag{1}$$

where $\mathbb{I}_2$ is identity operator in two dimensions, $\vec{u} = \mathrm{Tr}_A[\mathrm{M}_{AB}(\vec{\sigma}\otimes\mathbb{I}_2)]$ and $\vec{v} = \mathrm{Tr}_B[\mathrm{M}_{AB}(\mathbb{I}_2\otimes\vec{\sigma})]$ belong to real vectors, $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ with σ_i being Pauli operator, β is a real matrix with the entry $\beta_{i,j} = \mathrm{Tr}_{AB}[\mathrm{M}_{AB}(\sigma_i\otimes\sigma_j)]$. According to the equation $\mathrm{U}(\vec{r}\cdot\vec{\sigma})\mathrm{U}^\dagger = (\mathrm{O}\vec{r})\cdot\vec{\sigma}$ ($\mathrm{U}\in\mathrm{SU}(2)$ and $\mathrm{O}\in\mathrm{SO}(3)$), M_{AB} is equivalent to the form, up to local unitary operations

$$\rho_{AB} = \frac{1}{4}\left(\mathbb{I}_2\otimes\mathbb{I}_2 + \vec{m}\cdot\vec{\sigma}\otimes\mathbb{I}_2 + \mathbb{I}_2\otimes\vec{n}\cdot\sigma + \sum_{i=1}^{3}\alpha_i\sigma_i\otimes\sigma_i\right), \tag{2}$$

where $\vec{m} = \mathrm{O}_1\vec{u}$, $\vec{n} = \mathrm{O}_2\vec{v}$, $\alpha = \mathrm{diag}(\alpha_1, \alpha_2, \alpha_3) = \mathrm{O}_1\beta\mathrm{O}_2^+$. Since ρ_{AB} is locally equivalent to M_{AB}, for simplicity, only the type of the states Eq. (2) is considered in this Letter.

As shown in Refs. [1,9], any two-qubit unitary operation can be decomposed into local and completely nonlocal parts

$$\mathrm{U} = (\mathrm{A}_1\otimes\mathrm{B}_1)\exp\left(-i\sum_{i=1}^{3}\theta_i\sigma_i\otimes\sigma_i\right)(\mathrm{A}_2\otimes\mathrm{B}_2), \tag{3}$$