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Assisted cloning of an unknown two-particle entangled state

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Abstract

We propose a protocol where one can realize quantum cloning of an unknown two-particle entangled state and its orthogonal complement state with assistance offered by a state preparer. The first stage of the protocol requires usual teleportation via two entangled particle pairs as quantum channel. In the second stage of the protocol, with the assistance (through a two-particle projective measurement) of the preparer, the perfect copies and complement copies of an unknown state can be produced. We also put forward a scheme for the teleportation by using non-maximally entangled quantum channel. The clones and complement clones of unknown state can be obtained with certain probability in the latter scheme.

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Quantum information theory has opened up the possibility of novel form of information processing tasks which are not possible classically. Some important quantum information processing tasks have been quantum teleportation [1], quantum dense coding [2], quantum computation [3], quantum key distribution [4], and so on. In recent years, the possibility of cloning quantum states approximately has attracted much attention. A quantum state cannot be cloned exactly because of the no-cloning theorem [5,6]. However, quantum cloning approximately is necessary in quantum information [7]. Though exact cloning is no possible, in the literature various cloning machines have been proposed [8–22] which operate either in a deterministic or probabilistic way. Universal quantum cloning machines was originally addressed by Bužek and Hillery [10]. The deterministic state-dependent cloning machine, proposed firstly by Hillery and Bužek [11], is designed to generate approximate clones of states belonging to a finite set. Gisin and Massar [12] and Bruß et al. [13] constructed the universal cubit cloner that maximizes the local fidelity. The probabilistic cloning machine was first considered by Duan and Guo [14] using a general

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unitary-reduction operation with a postelection of the measurement results. Murao et al. [15] proposed the quantum telecloning process combining quantum teleportation and optimal quantum cloning from one input to M outputs. The other category of quantum cloning machines were developed by some authors [16–20].

Recently, Pati [16] proposed a scheme where one can produce perfect copies and orthogonal-complement copies of an arbitrary unknown state with minimal assistance from a state preparer. More recently, Chen and Wu [21] presented a protocol to probabilistically clone an unknown state and its orthogonal complement state with assistance. In this Letter, we propose a protocol which can produce copies of an unknown two-particle entangled state via two entangled particle pairs as the quantum channel. Different from the previous protocol using a single-particle von Neumann orthogonal measurement [16], here we will realize the assisted cloning by using a two-particle projective measurement consisting of a set of non-maximally entangled basis vectors.

Suppose Alice has an unknown input two-particle entangled state $|\phi\rangle_{12} = \alpha|00\rangle_{12} + \beta|11\rangle_{12}$ from a state preparer Victor, with α as a real number and β as a complex number, and $|\alpha|^2 + |\beta|^2 = 1$. Alice wishes to create either a copy or an orthogonal copy of the unknown state $|\phi\rangle_{12}$ at her place with the assistance of Victor. Assume Alice and Bob share two entangled particle pairs, as the quantum channel, given by

$$|\psi\rangle_{34} = \frac{1}{\sqrt{2}}\big(|01\rangle + |10\rangle\big)_{34}, \tag{1}$$

$$|\psi\rangle_{56} = \frac{1}{\sqrt{2}}\big(|01\rangle + |10\rangle\big)_{56}. \tag{2}$$

Here, we assume that particles 3 and 5 belong to Alice while particles 4 and 6 belong to Bob. The input state $|\phi\rangle_{12}$ is unknown to both Alice and Bob. The initial state of the combined system is

$$|\Psi\rangle = |\phi\rangle_{12} \otimes |\psi\rangle_{34} \otimes |\psi\rangle_{56} = |\Psi_{(1)}\rangle + |\Psi_{(2)}\rangle + |\Psi_{(3)}\rangle + |\Psi_{(4)}\rangle, \tag{3}$$

where

$$\begin{aligned}|\Psi_{(1)}\rangle = \frac{1}{4}\big[&\big|\Phi^+\big\rangle_{13}\big|\Phi^+\big\rangle_{25}\big(\alpha|11\rangle + \beta|00\rangle\big)_{46} + \big|\Phi^+\big\rangle_{13}\big|\Phi^-\big\rangle_{25}\big(\alpha|11\rangle - \beta|00\rangle\big)_{46} \\ &+ \big|\Phi^-\big\rangle_{13}\big|\Phi^+\big\rangle_{25}\big(\alpha|11\rangle - \beta|00\rangle\big)_{46} + \big|\Phi^-\big\rangle_{13}\big|\Phi^-\big\rangle_{25}\big(\alpha|11\rangle + \beta|00\rangle\big)_{46}\big],\end{aligned} \tag{4}$$

$$\begin{aligned}|\Psi_{(2)}\rangle = \frac{1}{4}\big[&\big|\Phi^+\big\rangle_{13}\big|\Psi^+\big\rangle_{25}\big(\alpha|10\rangle + \beta|01\rangle\big)_{46} + \big|\Phi^+\big\rangle_{13}\big|\Psi^-\big\rangle_{25}\big(\alpha|10\rangle - \beta|01\rangle\big)_{46} \\ &+ \big|\Phi^-\big\rangle_{13}\big|\Psi^+\big\rangle_{25}\big(\alpha|10\rangle - \beta|01\rangle\big)_{46} + \big|\Phi^-\big\rangle_{13}\big|\Psi^-\big\rangle_{25}\big(\alpha|10\rangle + \beta|01\rangle\big)_{46}\big],\end{aligned} \tag{5}$$

$$\begin{aligned}|\Psi_{(3)}\rangle = \frac{1}{4}\big[&\big|\Psi^+\big\rangle_{13}\big|\Phi^+\big\rangle_{25}\big(\alpha|01\rangle + \beta|10\rangle\big)_{46} + \big|\Psi^+\big\rangle_{13}\big|\Phi^-\big\rangle_{25}\big(\alpha|01\rangle - \beta|10\rangle\big)_{46} \\ &+ \big|\Psi^-\big\rangle_{13}\big|\Phi^+\big\rangle_{25}\big(\alpha|01\rangle - \beta|10\rangle\big)_{46} + \big|\Psi^-\big\rangle_{13}\big|\Phi^-\big\rangle_{25}\big(\alpha|01\rangle + \beta|10\rangle\big)_{46}\big],\end{aligned} \tag{6}$$

$$\begin{aligned}|\Psi_{(4)}\rangle = \frac{1}{4}\big[&\big|\Psi^+\big\rangle_{13}\big|\Psi^+\big\rangle_{25}\big(\alpha|00\rangle + \beta|11\rangle\big)_{46} + \big|\Psi^+\big\rangle_{13}\big|\Psi^-\big\rangle_{25}\big(\alpha|00\rangle - \beta|11\rangle\big)_{46} \\ &+ \big|\Psi^-\big\rangle_{13}\big|\Psi^+\big\rangle_{25}\big(\alpha|00\rangle - \beta|11\rangle\big)_{46} + \big|\Psi^-\big\rangle_{13}\big|\Psi^-\big\rangle_{25}\big(\alpha|00\rangle + \beta|11\rangle\big)_{46}\big],\end{aligned} \tag{7}$$

where $|\Phi^\pm\rangle_{ij}$ and $|\Psi^\pm\rangle_{ij}$ are the Bell states of particles i and j

$$\big|\Phi^\pm\big\rangle_{ij} = \frac{1}{\sqrt{2}}\big(|00\rangle \pm |11\rangle\big)_{ij}, \qquad \big|\Psi^\pm\big\rangle_{ij} = \frac{1}{\sqrt{2}}\big(|01\rangle \pm |10\rangle\big)_{ij}. \tag{8}$$

Assume Alice performs Bell-state measurements on particles $(1,3)$ and $(2,5)$, respectively, and if the measurement outcome of Alice is $|\Psi^-\rangle_{13}|\Psi^+\rangle_{25}$ (the probability of this result is only $1/16$), then the resulting six-particle state can be written as

$$\big|\Psi^+\big\rangle_{25}\big\langle\Psi^+\big|\Psi^-\big\rangle_{13}\big\langle\Psi^-\big|\Psi\big\rangle = \frac{1}{4}\big|\Psi^-\big\rangle_{13}\big|\Psi^+\big\rangle_{25}\big(\alpha|00\rangle - \beta|11\rangle\big)_{46}. \tag{9}$$