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Global exponential stability and periodicity of cellular neural networks with variable delays ☆

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Abstract

The Letter presents sufficient conditions ensuring the global exponential stability and existence of the periodic solution for cellular neural networks with variable delays. The results allow for the consideration of all unbounded neuron activation functions (but not necessarily surjective), in particular, can analyze the exponential stability and periodicity for the linear cellular neural networks. The work provides one such method which can be applied to cellular neural networks systems with variable delays. The method, based on the theory of fixed point and differential inequality technique. The applicability of the present results is demonstrated by two examples.

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Keywords: Global exponential stability; Periodic solution; Cellular neural network; Differential inequality; Fixed point

1. Introduction

Cellular neural network (CNN), as one of the most popular models in the literature of neural networks, was first introduced by Chua and Yang [1,2] in 1988. The CNN is formed by the basic circuit units called cells. A cell contains linear and nonlinear circuit elements, which typically are linear capacitors, linear resistor, linear and non-linear controlled sources, and independent sources. Its structure is analogous with that found in cellular automata, that is to say, any cell in a cellular neural networks is connected only to its neighbor cells. The circuit diagram

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and connection pattern implementing for the CNN can be referred to [1,2]. The dynamical characteristic such as stability and periodicity of the CNN play an important role in the image processing, pattern recognition and associative memories. As a consequence, many researchers have forced their attention on the study of stability and periodicity of the CNN with constant delays and without delays [3–22]. To the best of our knowledge, few authors discuss stability and periodicity of the CNN with variable delays [23,24]. In the practical applications, the delays are usually time varying, and sometimes vary violently with time due to the finite switching speed of amplifiers and faults in the electrical circuit. Moreover, in the proofs of stability and periodicity, we find many authors made use of the boundedness of the output of the cell functions. Unfortunately, the assumption makes the results which are not applicable to some important engineering and physical problems. For example, this is the case of CNN for solving optimization problems in the presence of constraint (linear or more general programming problems [25,26]).

Motivated by the above discussion, our objective in this Letter is to study further the global exponential stability and periodicity of the CNN with variable delays, and drop the assumption of the boundedness for the output of the cell functions. Here, we point out that our approach, which is different from previously known results, based on the properties of nonnegative matrices [27], the fixed point theory, and differential inequality technique [10,11,28].

The rest of the Letter is organized as follows. In Section 2, some notations and lemma are given. In Sections 3 and 4, we derive some sufficient conditions for the global exponential stability and the existence of periodicity for the CNN with variable delays. In Section 5, two examples are given to demonstrate the validity of our results. The conclusions are drawn in Section 6.

2. Notations and preliminary

Let $R^+ = [0, +\infty)$. $C[X, Y]$ is a continuous mapping set from the topological space X to the topological space Y. Especially, $C \triangleq C[[-r, 0], R^n]$.

We consider cellular neural networks system with variable delays as follows:

$$\begin{cases} \dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^n a_{ij} f_j(x_j(t)) + \sum_{j=1}^n b_{ij} g_j(x_j(t - r_j(t))) + I_i(t), & t \geqslant 0, \\ x_i(t) = \varphi_i(t), & -r \leqslant t \leqslant 0, \end{cases} \tag{1}$$

where n denotes the number of units in a neural network; $x_i(t)$ corresponds to the state of the ith unit at time t; $c_i > 0$ represents the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the network and external inputs; f_j and g_j denote the output of the j unit at time t and $t - r_j(t)$, respectively; a_{ij} denotes the strength of the jth unit on the ith unit at time t; b_{ij} denotes the strength of the jth unit on the ith unit at time $t - r_j(t)$; $I_i(t)$ represents the external bias on the ith unit at time t and $I_i(t): R^+ \to R$, is continuous; $r_j(t)$ denotes the transmission delay along the axon of the jth unit and is continuous with $0 \leqslant r_j(t) \leqslant r$.

For any $\varphi = \operatorname{col}\{\varphi_i\} \in C$, a solution of (1) is a function $x(t) = \operatorname{col}\{x_i(t)\}: R^+ \to R^n$ satisfying (1) for $t \geqslant 0$. Throughout the Letter, we always assume that system (1) has a continuous solution denoted by $x(t, 0, \varphi)$ or simply $x(t)$ if no confusion should occur.

For convenience in the following we shall rewrite Eq. (1) in the form

$$\begin{cases} \dot{x}(t) = -cx(t) + Af(x(t)) + Bg(x(t - r(t))) + I(t), & t \geqslant 0, \\ x(t) = \varphi(t), & -r \leqslant t \leqslant 0, \end{cases} \tag{2}$$

where $x(t) = \operatorname{col}\{x_i(t)\}$, $c = \operatorname{diag}\{c_i\}$, $A = (a_{ij})_{n\times n}$, $B = (b_{ij})_{n\times n}$, $f = \operatorname{col}\{f_i\}$, $g = \operatorname{col}\{g_i\}$, $I(t) = \operatorname{col}\{I_i(t)\}$, $\varphi(t) = \operatorname{col}\{\varphi_i(t)\}$.

Recall that the spectral norm of matrix M is defined as

$$\|M\|_2 = \left(\max\left\{\lambda\colon \lambda \text{ is an eigenvalue of } M^T M\right\}\right)^{1/2},$$