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Quantum electrodynamic foundations of continuum electrodynamics

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Abstract

A microscopic quantum electrodynamic model of a linear medium consisting of discrete quantized electric and magnetic dipoles is used to derive Maxwell equations of motion for macroscopic fields. It is found that the material properties of a linear medium are carried in the sum of the vacuum, electric, and magnetic susceptibilities.

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The electrodynamics of ponderable media is, at an atomistic level, the interaction of an electromagnetic field in the vacuum interacting with the individual particles of matter, generating reaction fields [1, 2]. Quantum electrodynamics describes this interaction microscopically in terms of the interaction of the quantized vacuum field with the individual quantized constituents of matter, while the classical theory of electric and magnetic fields in matter is based on experimental studies of macroscopic, averaged fields in continuous matter.

In this Letter, classical equations of motion for macroscopic electric and magnetic fields in a continuous linear medium are derived from the fully microscopic quantum electrodynamic theory. Beginning with a microscopic quantum electrodynamic model of a linear medium in terms of the quantized vacuum field and quantized electric and magnetic oscillators, the medium is eliminated, within the formalism of quantum electrodynamics, in the good-cavity limit [3]. Applying the adiabatic-following and continuum approximations produces an effective macroscopic quantum Hamiltonian [4]. Inverting the Ginzburg macroscopic quantization procedure [5–7], we obtain the analogous classical effective Hamiltonian, identify the bare, or fundamental, electric and magnetic fields in a linear medium, and derive equations of motion for the clas-

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sical fields. Retaining both the dielectric and magnetic properties of a general linear medium, we find that the inverse index of refraction appears as a linear combination of the vacuum, electric, and magnetic susceptibilities. Then, the phase velocity of light in a linear medium cn^{-1} is a linear function of the susceptibilities whereas the product of the electric permittivity ε and magnetic permeability μ is nonlinear in the electric and magnetic susceptibilities and is therefore incongruous with a linear medium. Consequently, the Maxwell equations

$$\frac{c}{n}\nabla\times\frac{n}{c}\mathbf{H}'-\frac{n}{c}\frac{\partial\mathbf{E}'}{\partial t}=\mathbf{0}, \tag{1a}$$

$$\nabla\times\mathbf{E}'+\frac{n}{c}\frac{\partial\mathbf{H}'}{\partial t}=\mathbf{0}, \tag{1b}$$

for the bare fields $\mathbf{E}'=-c^{-1}\partial\mathbf{A}/\partial t$ and $\mathbf{H}'=n^{-1}\nabla\times\mathbf{A}$ that are derived here have no explicit dependence on the electric permittivity ε or magnetic permeability μ and the permittivity and permeability of free space no longer separate the formulations of the Maxwell equations in SI and Gaussian cgs units.

The linear medium is regarded microscopically as being a mixture of discrete enumerated atom-like electric dipoles and spin-like magnetic dipoles and each electric and magnetic dipole is located by an independent position vector. The Hamiltonian for the model linear medium is

$$\begin{aligned}H=&\sum_{l\lambda}\hbar\omega_l a_l^\dagger a_l+\sum_n\hbar\omega_b b_n^\dagger b_n+\sum_m\hbar\omega_c c_m^\dagger c_m\\&-i\hbar\sum_{nl\lambda}\left(h_l a_l b_n^\dagger e^{i\mathbf{k}_l\cdot\mathbf{r}_n}-h_l^* a_l^\dagger b_n e^{-i\mathbf{k}_l\cdot\mathbf{r}_n}\right)\\&-i\hbar\sum_{ml\lambda}\left(f_l a_l c_m^\dagger e^{i\mathbf{k}_l\cdot\mathbf{r}_m}+f_l^* a_l^\dagger c_m e^{-i\mathbf{k}_l\cdot\mathbf{r}_m}\right),\end{aligned} \tag{2}$$

where $a_l^\dagger$ and a_l are the creation and destruction operators for field modes, ω_l is the frequency of the field in the mode l, $b_n^\dagger$ and b_n are the creation and destruction operators for the electric dipole at the fixed position $\mathbf{r}_n$, $c_m^\dagger$ and c_m are the creation and destruction operators for the magnetic dipole at the fixed position $\mathbf{r}_m$, ω_b is the resonance frequency associated with the electric dipoles, ω_c is the resonance frequency associated with the magnetic dipoles,

$$h_l=\left(\frac{2\pi\omega_l}{\hbar V}\right)^{1/2}\hat{\boldsymbol{\mu}}_e\cdot\hat{\mathbf{e}}_{\mathbf{k}_l\lambda}$$

is the electric dipole interaction in terms of the electric dipole moment $\boldsymbol{\mu}_e$, and

$$f_l=\left(\frac{2\pi\omega_l}{\hbar V}\right)^{1/2}\hat{\boldsymbol{\mu}}_m\cdot(\hat{\mathbf{k}}\times\hat{\mathbf{e}}_{\mathbf{k}_l\lambda})$$

is the magnetic dipole interaction in terms of the magnetic dipole moment $\boldsymbol{\mu}_m$. In order to condense the notation, the operator subscript λ for sums over the polarizations is combined with the mode index l. The usual dipole and rotating-wave approximations have been employed and zero-point energies have been suppressed. The dipole and rotating-wave approximations are used extensively in quantum optics and their characteristics and limitations are well known. As no sources are present, we work in the Coulomb gauge.

Microscopic Heisenberg equations of motion for the material and field mode operators

$$\begin{aligned}\frac{da_l}{dt}=&-i\omega_l a_l\\&+\sum_n h_l^* b_n e^{-i\mathbf{k}_l\cdot\mathbf{r}_n}+\sum_m f_l^* c_m e^{-i\mathbf{k}_l\cdot\mathbf{r}_m},\end{aligned} \tag{3a}$$

$$\frac{db_n}{dt}=-i\omega_b b_n-\sum_{l\lambda}h_l a_l e^{i\mathbf{k}_l\cdot\mathbf{r}_n}, \tag{3b}$$

$$\frac{dc_m}{dt}=-i\omega_c c_m-\sum_{l\lambda}f_l a_l e^{i\mathbf{k}_l\cdot\mathbf{r}_m}, \tag{3c}$$

are readily obtained from the Hamiltonian (2).

In the continuum electrodynamics of linear dielectrics, the medium is eliminated with the contribution of the medium carried as a constant of proportionality onto the basic macroscopic fields. The analogous elimination of the medium can be performed within the formalism of quantum electrodynamics by eliminating the equations of motion for the oscillators in the good-cavity limit [3,4]. Substituting the formal integral of Eqs. (3b) and (3c) into Eq. (3a) one obtains

$$\begin{aligned}\frac{da_l}{dt}=&-i\omega_l a_l+\sum_n h_l^* b_n(t_0)e^{-i\omega_b(t-t_0)}e^{-i\mathbf{k}_l\cdot\mathbf{r}_n}\\&+\sum_m f_l^* c_m(t_0)e^{-i\omega_c(t-t_0)}e^{-i\mathbf{k}_l\cdot\mathbf{r}_m}\\&-\sum_{nl'\lambda'}h_l^* h_{l'}e^{-i(\mathbf{k}_l-\mathbf{k}_{l'})\cdot\mathbf{r}_n}\int_{t_0}^{t}dt'\,e^{-i\omega_b(t-t')}a_{l'}(t')\end{aligned}$$