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On supercorrelated systems and phase space entrainment

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Abstract

It is demonstrated that power-laws which are modified by logarithmic corrections arise in supercorrelated systems. Their characteristic feature is the energy attributed to a state (or value of a general cost function) which depends nonlinearly on the phase space distribution of the constituents. A one-dimensional dissipative deterministic model is introduced which is attracted to a supercorrelated state (phase space entrainment). Extensions of this particular model may have applications in the study of transport and equilibration phenomena, particularly for supply and information networks, or for chemical and biological nonequilibrium systems, while the qualitative arguments presented here are believed to be of more general interest.

Keywords: Power-law; Correlation; Nonequilibrium system; Statistical mechanics

1. Introduction

Power-laws are omnipresent in natural or man-made systems [1,2]. They arise in the areas of high-energy particle physics, condensed matter, complexity, sociology, and linguistics, to name a few. They are an important feature of Tsallis statistics [3,4]. While numerous out-of-equilibrium statistical systems are known showing this behavior, only rarely the dynamics is understood that leads to it.

In this Letter we study systems that are correlation dominated, in a sense to be more accurately defined. We show that in this class of models the asymptotic spectra are essentially given by power-laws, however, modified by logarithmic corrections. They appear quite similar from a phenomenological point of view.

Recently, for example, most interesting transient dynamical behavior has been observed in relativistic heavy-ion collisions [5]. It is observed that apparently thermodynamics rules, with a local equation of state, in particular, and commencing at unexpectedly early times (of order 0.3–0.9 $\times 10^{-24}$ s) during the collision process. However, the experiments also indicate that typical one-particle observables are not

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correspondingly thermalized by that time. Theoretical investigations based on quantum field theoretical models support the view that such 'prethermalization' is indeed produced [6]. This is accompanied by spectra, which show considerable deviations from Fermi–Dirac, Bose–Einstein, or Boltzmann distributions, with uniform parameters, and which relax to the equilibrium form only on a much longer time scale.

However, a heuristic picture which allows us to understand essential features of the underlying processes generally has been lacking. Recently, we have shown that such behavior can be interpreted in terms of certain correlations generated dynamically and, in fact, power-laws can emerge as a consequence [7]. Presently, we start with a different class of models. In some limits analytical results can be obtained, the models can be easily generalized, and may be applied in quite different contexts. We demonstrate the emergence of log-modified power-laws as a robust feature in *supercorrelated systems* where correlations among the distributions of the constituents govern the statistical behavior.

Other recent works invariably invoke stochastic processes in the explanation of power-laws, e.g., in Refs. [7–11]. In distinction, we present a deterministic dissipative model where microscopic stochastic processes are subsumed in suitable (anti)damping terms of otherwise Hamiltonian equations of motion. We demonstrate that *phase space entrainment* leads to supercorrelation here and, thus, ultimately to the log-modified power-laws.

2. Supercorrelated systems

Specifically, we consider a two-dimensional 'phase space' spanned by a discrete 'spatial' coordinate $x \in \{x_i,\ i = 1, \ldots, L\}$ (with periodic boundary condition) together with a discrete 'energy' coordinate $E \in \{E_j,\ j = 1, \ldots, M\}$. We would like to understand the statistical behavior of a system consisting of L 'particles', which are distributed randomly over the energy E, however, with exactly one particle per lattice site x.

The interpretation assigned to the coordinates may vary according to circumstances. In particular, the 'energy' may generally represent any *cost function*.

Crucially, we assume that there is a characteristic energy E_j associated with the jth state of the system. Furthermore, it is the product of this energy times a correlation function C_j which yields the relevant effective energy. Only the latter will determine the relative weight of terms in a sum over states. More precisely, the correlation function $C_j[p]$ is determined by a product of particle distributions p. The number of factors of p involved will be called the correlation degree c. (Out of a sum of terms of different degree only the one with smallest c matters asymptotically.)

Before we proceed, we define this correlation energy in various examples:

- If the correlation is between a particle at site x and a second one a distance Δx away, then the contribution to the total energy is:
$$\sum_j E_j C_j[p] \equiv \sum_{i,j} E_j\, p(x_i, E_j)\, p(x_i + \Delta x, E_j). \tag{1}$$
Note that all possible pairs ($c = 2$) are counted here exactly once, due to the periodic boundary condition.
- Another case of interest is that the correlation of the particles does not depend on the mutual distance, corresponding to the limit that the interaction length is larger than the system size, $\Delta x > x_L - x_1$:
$$\sum_j E_j C_j[p] \equiv \sum_{i_1<i_2<\cdots<i_c;\, j} E_j\, p(x_{i_1}, E_j) \times p(x_{i_2}, E_j) \cdots p(x_{i_c}, E_j) \tag{2}$$
with c ordered factors of p, not restricted otherwise because of lattice periodicity.
- Finally, we may have an anticorrelation instead of the previous case:
$$\sum_j E_j C_j[p] \equiv \sum_{i_1;\, j} E_j\, p(x_{i_1}, E_j) \times \Bigl(1 - \sum_{i_1<i_2<\cdots<i_c} p(x_{i_2}, E_j) \cdots p(x_{i_c}, E_j)\Bigr). \tag{3}$$
Numerous further variants can be imagined.

It seems worth while to discuss various features here. First of all, it is important to realize that the energies E_j need neither be related to single-particle nor to