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Even harmonics generation in plasma as a new tool of current evolution diagnostics

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Abstract

It is shown that using even harmonics of a test wave it is possible to diagnose the fast time evolution of the current density.

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Electron collisions with ions represent one of the basic physical mechanism yielding in a plasma the generation of harmonics of a high-frequency (HF) field. For a plasma with an initial Maxwellian electron distribution function (EDF) the generation of odd harmonics has been predicted many years ago [1]. The selective generation of only odd harmonics takes place when the EDF is invariant with respect to electron velocity direction inversion [2–4]. Such invariance property does not hold, if in plasma the electron fluxes are present (see, for instance, [5,6]). As a consequence, in a current-carrying plasma [7], or in a plasma with heat flux [8], besides odd harmonics generation, the even harmonics generation as well takes place. As electron fluxes are the sources of even harmonics generation, the detailed investigation of the latter may serve as the basis to obtain information on the electron fluxes in a plasma. In this communication, we investigate the flux density time evolution of radiation at the even harmonics frequencies of a test wave, and establish its link with the evolution of the current density produced by a constant electric field. We show how the generation efficiency of the second and forth harmonics is correlated to the current density values. The characteristics concerning even harmonics generation, established below, may serve as the basis of a new diagnostic method to study plasma electron conductivity in situations when small time and space resolutions are required.

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Let us consider a fully ionized plasma in the presence of a uniform constant electric field $\vec{E}_0$. We assume that the ion ionization multiplicity Z is greater than unity and that the electric field strength E_0 is much less than the strength of the Dreicer critical field $E_D = m v_T \nu |e|^{-1}$,

$$E_0 \ll \frac{\sqrt{2\pi}}{32\sqrt{Z}} E_D, \tag{1}$$

where $\nu = \nu(v_T) = 4\pi Z e^4 N \Lambda m^{-2} v_T^{-3}$ is the electron–ion collision frequency, Λ is the Coulomb logarithm, e, m, N and v_T are, respectively, the electron charge, mass, density and thermal velocity. If the electric field satisfies the condition (1), then the electron distribution weakly deviates of the Maxwellian $F_m(v) = N(2\pi)^{-3/2} v_T^{-3} \exp(-v^2/2v_T^2)$. Assuming that the constant electric field is switched on at the moment $t = 0$ one can write the following weakly anisotropic EDF

$$F(\vec{v}) = F_m(v) - \frac{e}{m\nu(v)}\{1 - \exp[-\nu(v)t]\}\left(\vec{E}_0 \frac{\partial}{\partial \vec{v}}\right) F_m(v). \tag{2}$$

The current density corresponding to the distribution (2) is

$$\vec{j}(t) = e\int d\vec{v} \cdot \vec{v} F(\vec{v}) = \frac{e^2 N}{m\nu} \zeta(\nu t) \vec{E}_0, \tag{3}$$

where the time dependency is given by the function

$$\zeta(\tau) = \frac{2}{3\sqrt{2\pi}}\left[48 - \int_0^\infty du\, u^7 \exp\left(-\frac{u^2}{2} - \frac{\tau}{u^3}\right)\right]. \tag{4}$$

The plot of $\zeta(\tau)$ is shown in Figs. 1 and 2. Below, EDF (2) is used to investigate harmonics generation of HF radiation acting on a current-carrying plasmas. Assuming that the HF field switching-on time is much smaller than the time of EDF (2) modification, the HF field may be written as $\vec{E}\cos(\omega t - \vec{k}\vec{r})$ where $\vec{k} \cdot \vec{E} = 0$, and the frequency ω is linked to the wave vector $\vec{k}$ by the dispersion relation $\omega^2 = \omega_L^2 + k^2c^2 \simeq k^2c^2$, $\omega_L^2 = 4\pi e^2 N/m$. We assume that the amplitude of the electron quiver velocity $v_E = |\vec{v}_E| = |e\vec{E}/m\omega|$ is much smaller than the speed of light c. It will permit to describe the HF field action on the plasma within the dipole approximation, neglecting both the EDF spatial variations and the radiation magnetic field influence on the electrons. As the quasistationary electric field $\vec{E}_0$ is weak (see (1)), and the electron–ion collision frequency much smaller than ω, the EDF in the HF field may be represented as $f = f_0 + \delta f$, where δf is a small addition, while $f_0 = F[\vec{v} - \vec{v}_E(t)]$, with $\vec{v}_E(t) = \vec{v}_E \sin(\omega t - \vec{k}\vec{r})$. The equation for the small addition δf has the form

$$\frac{\partial}{\partial t}\delta f + \frac{e}{m}\vec{E}_0 \frac{\partial}{\partial \vec{v}}\delta f + \frac{e}{m}\vec{E}\cos(\omega t - \vec{k}\vec{r})\frac{\partial}{\partial \vec{v}}\delta f = -\frac{e}{m}\vec{E}_0\frac{\partial}{\partial \vec{v}} f_0 + \frac{1}{2}\nu(v)\frac{\partial}{\partial v_i}\left(v^2\delta_{ij} - v_i v_j\right)\frac{\partial}{\partial v_j} f_0. \tag{5}$$

The approximate Eq. (5) is valid for δf changing with the frequency ω or with greater frequencies. As the EDF (2) fully defines the electron density N, the correction δf satisfies the condition $\int d\vec{v}\, \delta f = 0$. Taking into account this condition, from (5) we obtain the time derivative of the HF correction to the current density $\delta\vec{j} = e\int d\vec{v} \cdot \vec{v}\delta f$. The time derivative of $\delta\vec{j}$ allows to find the correction to the current density brought about by the electron–ion collisions at the fundamental frequency, and the current density of high order harmonics as well. The term in $\partial\delta\vec{j}/\partial t$, which does not contain the static field $\vec{E}_0$, describes the generation of odd harmonics. The theory of HF field odd harmonics generation in a fully ionized plasma is developed in Ref. [1]. The term in $\partial\delta\vec{j}/\partial t$ containing the field $\vec{E}_0$ describes instead the current density leading to even harmonics generation. This term may be represented as

$$\frac{\partial}{\partial t}\delta\vec{j}_{\text{even}} = \sum_{n=1}^{\infty}\left(\frac{\partial}{\partial t}\delta\vec{j}\right)_{2n}\cos[2n(\omega t - \vec{k}\vec{r})]. \tag{6}$$

In the general case, the expression for $(\partial\delta\vec{j}/\partial t)_{2n}$ has components along and perpendicularly to the HF radiation wave vector. Current oscillations along $\vec{k}$ are purely potential and for the harmonics generation are not of interest. Current oscillations in the plane perpendicular to $\vec{k}$ are, instead, the sources of e.m. radiation. Being interested in even harmon-