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Global exponential stability of BAM neural networks with distributed delays and reaction–diffusion terms

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Abstract

Two sufficient conditions, which ensure the existence and uniqueness of the equilibrium point and global exponential stability of bi-directional associative memory (BAM) neural networks with distributed delays and reaction–diffusion terms, are obtained by using the theory of topological degree, properties of M-matrix, Lyapunov functional, and analysis technique. The two sufficient conditions are independent of each other. The results remove the usual assumption that the activation functions are of bounded character. Exponential converging velocity index is estimated, which depends on the delay kernel functions and system parameters. Two numerical examples are given to show the correctness of our analysis. These results can be applied to design globally exponentially stable networks and thus have important significance in both theory and applications.

Keywords: Bi-directional associative memory (BAM); Distributed delays; Reaction–diffusion terms; Global exponential stability; Topological degree; M-matrix; Lyapunov functional

1. Introduction

A class of neural networks related to bidirectional associative memory (BAM) has been proposed by Kosko [1–4]. These models generalize the single-layer autoassociative Hebbian correlator to a two-layer pattern-matched heteroassociative circuits. Therefore, this class of networks has good application perspective in the fields of pattern recognition and artificial intelligence [5]. Kosko has considered global stability of BAM models, his approach requires severe constrain conditions of having symmetric connection weight matrix. However, from the viewpoint of biological neural networks and their implementation on very large-scale integration or optical tech-

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nology, it is impossible that the connection weight matrix remains absolutely symmetric. Thus the stability analysis on BAM networks with asymmetric connection has been the highlight in those fields. As is well known, in both biological and man-made neural networks, the delays arise because of the processing of information. More specifically, in the electronic implementation of analog neural networks, the delays occur in the communication and response of neurons owing to the finite switching speed of amplifies [32–39]. Thus, the study of neural dynamics with consideration of the delayed problem becomes extremely important to manufacture high quality neural networks. Unfortunately, most of the existing investigations on BAM neural networks with delays were restricted to pure-delay models [6–19]. Recently, Liao et al. [20] investigated the stability of hybrid BAM neural networks with distributed delays. However, strictly speaking, diffusion effects cannot be avoided in the neural networks when electrons are moving in asymmetric electromagnetic fields. So we must consider that the activations vary in space as well as in time. [21–26] have considered the stability of neural networks with diffusion terms, which are expressed by partial differential equations. It is also common to consider the diffusion effects in biological systems (such as immigration) [27–29]. To the best of our knowledge, this is the first paper to consider global exponential stability for the BAM neural networks with distributed delays and reaction–diffusion terms. In this Letter, we will derive some new and simple criteria of the global exponential stability and give the estimation of exponential converging index for BAM neutral networks with distributed delays and reaction–diffusion terms by constructing suitable Lyapunov functional and using some analytic techniques. The work will have significance impact on the design and applications of globally exponentially stable BAM neural networks with distributed delays and diffusion terms, and are of great interest in many applications.

2. Model description

In this Letter, we consider the following model:

$$\begin{cases} \frac{\partial u_i(t,x)}{\partial t} = \sum_{k=1}^{l} \frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial u_i}{\partial x_k}\right) - a_i u_i + \sum_{j=1}^{m} w_{ji} g_j(v_j) + \sum_{j=1}^{m} w_{ji}^* \int_{-\infty}^{t} K_{ji}(t-s) g_j\big(v_j(s,x)\big)\,ds + I_i, \\ \frac{\partial v_j(t,x)}{\partial t} = \sum_{k=1}^{l} \frac{\partial}{\partial x_k}\left(D_{jk}^*\frac{\partial v_j}{\partial x_k}\right) - b_j v_j + \sum_{i=1}^{n} h_{ij} f_i(u_i) + \sum_{i=1}^{n} h_{ij}^* \int_{-\infty}^{t} N_{ij}(t-s) f_i\big(u_i(s,x)\big)\,ds + J_j, \end{cases} \tag{1}$$

for $i \in \{1,2,\ldots,n\}$, $j \in \{1,2,\ldots,m\}$, $t > 0$, where $x = (x_1, x_2, \ldots, x_l)^{\mathrm{T}} \in \Omega \subset R^l$, Ω is a bounded compact set with smooth boundary $\partial\Omega$ and $\operatorname{mes}\Omega > 0$ in space R^l; $u = (u_1, u_2, \ldots, u_n)^{\mathrm{T}} \in R^n$, $v = (v_1, v_2, \ldots, v_m)^{\mathrm{T}} \in R^m$. $u_i(t,x)$ and $v_j(t,x)$ are the state of the ith neurons and the jth neurons at time t and in space x, respectively; f_i and g_j denote the signal functions of the ith neurons and jth neurons at time t and in space x, respectively; I_i and J_j denote the external inputs on the ith neurons and the jth neurons, respectively; $a_i > 0$, $b_j > 0$, w_{ji}, w_{ji}^*, h_{ij}, h_{ij}^* are constants, a_i and b_j denote the rate with which the ith neurons and the jth neurons will reset their potential to the resting state in isolation when disconnected from the networks and external inputs, respectively; w_{ji}, w_{ji}^*, h_{ij} and h_{ij}^* denote the connection weights. Smooth functions $D_{ik} = D_{ik}(t,x,u) \geqslant 0$ and $D_{jk}^* = D_{jk}^*(t,x,u) \geqslant 0$ correspond to the transmission diffusion operators along the ith neurons and the jth neurons, respectively.

The boundary conditions and initial conditions are given by

$$\begin{cases} \frac{\partial u_i}{\partial n} := \left(\frac{\partial u_i}{\partial x_1}, \frac{\partial u_i}{\partial x_2}, \ldots, \frac{\partial u_i}{\partial x_l}\right)^{\mathrm{T}} = 0, \quad i = 1,2,\ldots,n, \\ \frac{\partial v_j}{\partial n} := \left(\frac{\partial v_j}{\partial x_1}, \frac{\partial v_j}{\partial x_2}, \ldots, \frac{\partial v_j}{\partial x_l}\right)^{\mathrm{T}} = 0, \quad j = 1,2,\ldots,m, \end{cases} \tag{2}$$

and

$$\begin{cases} u_i(s,x) = \phi_{u_i}(s,x), \quad s \in (-\infty, 0], \quad i = 1,2,\ldots,n, \\ v_j(s,x) = \phi_{v_j}(s,x), \quad s \in (-\infty, 0], \quad j = 1,2,\ldots,m, \end{cases} \tag{3}$$

where $\phi_{u_i}(s,x)$, $\phi_{v_j}(s,x)$ $(i = 1,2,\ldots,n;\ j = 1,2,\ldots,m)$ are bounded and continuous on $(-\infty, 0] \times \Omega$.

We assume that the activation functions and delay kernels functions satisfy the following properties: