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Practical measurement of joint weak values and their connection to the annihilation operator

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Abstract

Weak measurements are a new tool for characterizing post-selected quantum systems during their evolution. Weak measurement was originally formulated in terms of von Neumann interactions which are practically available for only the simplest single-particle observables. In the present work, we extend and greatly simplify a recent, experimentally feasible, reformulation of weak measurement for multiparticle observables [Phys. Rev. Lett. 92 (2004) 130402]. We also show that the resulting "joint weak values" take on a particularly elegant form when expressed in terms of annihilation operators.

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Weak measurement was originally proposed by Aharonov, Albert and Vaidman (AAV) [1] as an extension to the standard von Neumann ("strong") model of measurement. A weak measurement can be performed by sufficiently reducing the coupling between the measuring device and the measured system. In this case, the pointer of the measuring device begins in a state with enough position uncertainty that any shift induced by the weak coupling is insufficient to distinguish between the eigenvalues of the observable in a single trial. While at first glance it may seem strange to desire a measurement technique that gives less information than the standard one, recall that the entanglement generated between the quantum system and measurement pointer is responsible for collapse of the wavefunction. Furthermore, if multiple trials are performed on an identically-prepared ensemble of systems, one can measure the average shift of the pointer to any precision—this average shift is called the weak value. A surprising characteristic of weak values is

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that they need not lie within the eigenvalue spectrum of the observable and can even be complex [2–4]. On the other hand, an advantage of weak measurements is that they do not disturb the measured system nor any other simultaneous weak measurements or subsequent strong measurements, even in the case of noncommuting observables. This makes weak measurements ideal for examining the properties and evolution of systems before post-selection and might enable the study of new types of observables. Weak measurements have been used to simplify the calculation of optical networks in the presence of polarization-mode dispersion [5], applied to slow- and fast-light effects in birefringent photonic crystals [6], and bring a new, unifying perspective to the tunneling-time controversy [7, 8]. Hardy's paradox, introduced in Ref. [9], was analyzed in terms of weak values in [10]. In Ref. [11], weak values were used to physically explain the results of the cavity QED experiment described in [12]. The opposing views expressed in Refs. [13,14] on the role of which-path information and the Heisenberg uncertainty principle in the double-slit experiment are reconciled with the use of weak values in [15]. Weak measurement can be considered the best estimate of an observable in a pre- and post-selected system [16].

The von Neumann interaction was originally used to model standard quantum measurement by mathematically describing the coupling between the measured system and the measurement pointer [17]. The interaction couples an observable $\hat{A}$ of the quantum system to the momentum $\hat{P}$ of the pointer,

$$\mathcal{H} = g\hat{A}\hat{P}, \tag{1}$$

where g is the coupling constant which is assumed to be real to keep $\mathcal{H}$ Hermitian. Since $\hat{A}$ and $\hat{P}$ act in different Hilbert spaces we can safely assume they commute. This interaction would be difficult to implement were it not for the fact that typically the measured system itself is used as part of the measurement device. When measuring $\hat{A}$ of a particle an independent degree of freedom of the particle can be used as the pointer. For example, a birefringent crystal can be oriented so that it will displace the position of photon by an amount that depends on the photon's polarization [18]. Here, $\hat{A}$ is the polarization observable and the pointer is the position of the photon. Another example is the Stern–Gerlach apparatus, where $\hat{A}$ is the spin of the particle and the pointer is the momentum of the particle. If such a measurement strategy were not available, one would require a strong controllable interaction between the quantum system and a separate pointer system. This is typically far too technically difficult to implement.

In modern quantum mechanics, we are increasingly interested in a different class of observables than in the above example, in which only a single particle is involved. Often, one would like to measure correlations between observables of distinct particles, like $\hat{S}_1\hat{S}_2$, the spin of particle one times the spin of particle two. Moreover, any experiment that utilizes or directly measures properties of entanglement is based on such observables and so, much of quantum information and quantum optics deal with these composite or joint observables. The exciting results and complex, rich range of features discovered by studies of entanglement suggests that weak measurement of joint observables should also produce valuable and interesting results. In fact, there already exist a few theoretical ideas for weak measurements that center around joint observables, such as Hardy's paradox [10], nonlocality of a single particle [8], and extensions of the quantum box problem [19,20]. We call the weak value of a joint observable the "joint weak value". If the composite observable is a product of N single-particle observables then the weak value is called the "Nth-order joint weak value".

Joint observables are extremely difficult to measure directly with either strong or weak types of measurement. The difficulty lies in the fact that the necessary von Neumann interaction couples two separate observables, and hence particles, to a single pointer. One, therefore, can no longer use the extra degree of freedom on one of the particles as the pointer and so, one requires multiparticle interactions. An approach using multiparticle interactions was outlined in a proposal for a weak measurement experiment with ions but so far there have been no experimental weak measurements of joint observables [21]. On the other hand, experimental strong measurements of joint observables are feasible and even commonplace. This is made possible by employing a different measurement strategy. Instead of measuring the joint observable directly, each single-particle observable is measured simultaneously but separately. For example, instead of measuring $\hat{S}_1\hat{S}_2$ directly we can measure $\hat{S}_1$ and $\hat{S}_2$ separately and then multiply the results trial by trial.