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Continuous time limits of repeated games with imperfect public monitoring

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Abstract

In a repeated game with imperfect public information, the set of equilibria depends on the way that the distribution of public signals varies with the players' actions. Recent research has focused on the case of "frequent monitoring," where the time interval between periods becomes small. Here we study a simple example of a commitment game with a long-run and short-run player in order to examine different specifications of how the signal distribution depends upon period length. We give a simple criterion for the existence of efficient equilibrium, and show that the efficiency of the equilibria that can be supported depends in an important way on the effect of the player's actions on the variance of the signals, and whether extreme values of the signals are "bad news" of "cheating" behavior, or "good news" of "cooperative" behavior.
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1. Introduction

In a repeated game with imperfect public information, the set of equilibria depends on the way that the distribution of public signals varies with the players' actions. When considering the case of "frequent monitoring," where the time interval between periods becomes small, it

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seems natural to suppose that the distribution of signals changes in some way as the time period shrinks. In this paper, we model the dependency of the information structure on the period length by supposing that there the players observe the state of a fixed continuous-time process at the start of each period, and that this process is either Poisson or a diffusion.

Intuitively, if the public signal is "sales" or "revenues," it corresponds to the aggregate of a number of individual transactions, so that over small enough time intervals we would observe at most a single transaction. Even for a monetary aggregate that measures all transactions in an economy, in any given picosecond we are unlikely to observe more than a single trade, so the discrete Poisson process, then, is one natural way to model the frequent observation of revenues. In practice, however, it is often not practical or possible to observe at a high enough frequency to track every discrete event. Instead, what is observed over the relevant time period is an aggregate of many events, and under standard conditions this aggregate converges to a diffusion as the period between events and their size both become small at a particular relative rate. The continuous-time limit we compute here thus corresponds to the iterated limit where the observation period of the players, though short, is much longer than the period between events.[1]

Our goal is to illuminate some conceptual points about the relationship between discrete and continuous time repeated games, and not to present a general theory, so we specialize throughout the paper to a specific example of a repeated game between a single long-run player and a sequence of short-run opponents. In this setting, the best equilibrium payoff can be attained by a "grim" strategy that prescribes the efficient outcome so long as the public signal above a critical threshold. Our first main result, Proposition 1, shows how the existence of efficient or non-trivial equilibria in the limit of time periods shrinking to zero can be determined by two properties of the limits of the probabilities p and q that punishment is triggered under the equilibrium action and defection, respectively. Specifically, the key variables are the limit of the signal-to-noise ratio $(q - p)/p$, which we denote by ρ, and the limit μ of the rate at which deviation increases transitions to the punishment regime $\mu = (q - p)/\tau$ where τ is the length of the period. We show that there is a non-trivial limit equilibrium if ρ is sufficiently large and $\mu > 0$, and that there is an efficient equilibrium in the iterated limit where first τ and then r go to 0 if $\rho = \infty$ and $\mu > 0$.

Proposition 1 applies for arbitrary specifications of how the signal structure depends on the period length; the remainder of this paper considers the case where the signals comes from observing an underlying Poisson or diffusion process. We find that the equilibrium set is larger (and so efficient outcomes are more likely to be supportable by equilibria) when the public signals correspond to the aggregation of a great many signals, that is, in the diffusion case, and that efficiency is less likely with Poisson signals. In addition, we find that when the signal is based on a diffusion what matters is the effect of the players' actions on the variance, of the aggregate, as opposed to its mean: Efficiency is more likely when the "tempting" or "cheating" actions generate a higher variance. Note that differential variances do not always result in full efficiency despite the fact that if the process is observed continuously the action would be known exactly. (Note also in a Poisson process (aggregated or not), the mean and variance are linked, so actions that increase the variance must increase the mean.) Our results show that the case where player's actions control the drift but not the variance of a diffusion process, is a knife-edge, at least when

[1] We examine more general ways of passing to the continuous time limit in a companion paper, Fudenberg and Levine (2007); this lets us explore the sensitivity of results about the diffusion case to the amount of "information aggregation" within each period.

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