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# Measuring scale effects in the allocative profit efficiency

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This paper provides an approach to the measurement of the "scale effects" in the allocative profit efficiency. To be specific, we evaluate the improvements of profit that can be accomplished by means of a change in the scale size, once technical efficiency is achieved. New decompositions of the allocative efficiency into a scale effect component and the corresponding residual mix effect component are derived.



## 1. Introduction

Ruiz and Sirvent [8] propose a slack-based "data envelopment analysis" (DEA) approach to the measurement of the technical component of the overall profit efficiency with the purpose of guaranteeing the achievement of the Pareto efficiency. This ensures that the allocative component, which is defined as the resulting residual, can be correctly estimated and reflects the profit improvements that can be accomplished by reallocations along the Pareto-efficient frontier. These authors define new measures of technical and allocative efficiency in terms of both profit ratios and differences of profits. They also extend their approach in order to derive individual lower and upper bounds for these efficiency components by using the novel models of minimum distance to the frontier in Aparicio et al. [2].

We noted in that paper that the movements from technically efficient points to the maximum profit point may involve changes in scale size, and it was left as future research the development, in the context of that approach, of a measure that accounts for these "scale effects" in the allocative profit efficiency. With a methodology similar to that used in Ruiz and Sirvent [8] in order to decompose the overall efficiency into technical and allocative components, we now propose new decompositions of the allocative efficiency into a component that measures the scale effect and a mix effect component, which is again defined as the corresponding residual. To be specific, the measures we define reflect profit improvements that can be accomplished by means of a change either in the scale size or in the mix of inputs and outputs, once technical efficiency is achieved.

In the present paper we thus assume that scale effects exist when a component of the profit improvement associated with the allocative efficiency can be explained by a change in the scale size. These scale effects should not be confused with scale inefficiency in the conventional sense of Banker [3], since they do not involve any comparison between constant returns to scale (CRS) and variable returns to scale (VRS) frontiers. In fact, since we assume VRS in our approach, we could even identify scale effects in the assessment of most productive scale size (MPSS) units, because in that case maximum profit units do not need to be MPSS units (i.e., maximum profit units do not need to be scale efficient in the conventional sense). Obviously, the scale effects in those cases would involve changes in the scale that would not be advisable from a pure technological perspective, although they can be justified in terms of profit improvements.

The decomposition of the allocative efficiency into components due to scale effects and mix effects is an issue that has been previously addressed in Portela and Thanassoulis [6]. As in ours, scale effects in the approach by Portela and Thanassoulis are not scale inefficiency either. Nevertheless, it should be noted that these authors deal with the adjustments involved in moving from the

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technical efficient projection to the maximum profit point, while the new decomposition we develop here is concerned with the profit improvements involved.

The present approach is of special interest in the for-profit sector, in particular because it considers explicitly information on prices. For-profit organizations are concerned with increasing profits, and assessing profit efficiency allows them to evaluate the actual production plans with respect to those that are profit maximizing. The measurement of the scale effects can complement the basic decomposition of the overall profit efficiency into the technical and allocative components with some information regarding changes in the scale that are advisable from the perspective of profit improvement.

The paper is organized as follows: In section 2 we briefly describe how overall profit efficiency is decomposed into technical and allocative components in Ruiz and Sirvent [8]. In section 3 we propose a decomposition of the allocative efficiency into scale and mix effects, which is illustrated with a small example in section 4. Last section concludes.

## 2. Background

This section summarizes the results in Ruiz and Sirvent [8] and thus provides the framework where the approach in the present paper is developed. Throughout the paper we consider a set of $n$ decision making units (DMUs), which use $m$ inputs to produce $s$ outputs. The so-called production possibility set (PPS), $T = \{(x, y) \in \mathbb{R}_+^{m+s} / \text{x can produce y}\}$, is empirically constructed from the observations by assuming the classical postulates of a DEA VRS technology (Banker [3]), so that it can be expressed as $T = \{(x, y) \in \mathbb{R}_+^{m+s} / x \geq X\lambda,\ y \leq Y\lambda, 1^t\lambda = 1,\ \text{for some}\ \lambda \geq 0\}$, where X is the $m \times n$ matrix with the input vectors and Y is the $s \times n$ matrix with the output vectors, and the intensities λ's represent the role of the different DMUs in each of the production plans in the PPS. Let $p_0 > 0$ be a $s \times 1$ vector with the prices of the $s$ outputs and $c_0 > 0$ a $m \times 1$ vector with the prices of the $m$ inputs of a given $DMU_0$.

The maximum profit that a given $DMU_0$ can achieve by operating in a fully efficient manner can be obtained as the optimal value of the following problem proposed in Färe et al. [5]:

$$\begin{aligned} \Pi_0^* = & \underset{x,y}{Max} \quad p_0^t y - c_0^t x \\ & s.t.: \\ & (x, y) \in T \end{aligned} \tag{1}$$

This maximum profit can be actually expressed as $\Pi_0^* = p_0^t y^* - c_0^t x^*$, where $(x^*, y^*)$ is the optimal solution of (1) and denotes the maximum profit point with the given prices.

A measure of the overall profit efficiency can be defined by comparing the actual profit of $DMU_0$, $\Pi_0 = p_0^t y_0 - c_0^t x_0$, with the maximum profit $\Pi_0^*$. To be precise, we can use either a ratio of profits $OE_0 = \dfrac{\Pi_0}{\Pi_0^*}$ (this implicitly assumes actual profits strictly positive) or a measure based on differences of profits $OE'_0 = \dfrac{\Pi_0^* - \Pi_0}{p_0^t y_0 + c_0^t x_0}$. Note that this latter measure $OE'_0$ coincides with the Nerlovian efficiency measure defined in Chambers et al. [4] in the particular case of using the observed inputs and outputs as the directional vector.

In a similar manner as with the overall profit efficiency measure, a measure of technical efficiency can be defined by comparing the actual profit of $DMU_0$, $\Pi_0$, with the maximum profit that this unit could achieve operating in a technically efficient manner, $\Pi_0^T$. Ruiz and Sirvent [8] obtain this profit $\Pi_0^T$ as the optimal value of the following LP problem they propose, which determines the maximum profit that $DMU_0$ could achieve by reducing its inputs and/or expanding its outputs:

$$\begin{aligned} \Pi_0^T = & \underset{s_0^{T-}, s_0^{T+}}{Max} \quad p_0^t (y_0 + s_0^{T+}) - c_0^t (x_0 - s_0^{T-}) \\ & s.t.: \\ & (x_0 - s_0^{T-}, y_0 + s_0^{T+}) \in T \\ & s_0^{T-} \geq 0_m, s_0^{T+} \geq 0_s \end{aligned} \tag{2}$$

An important feature of this model is that it guarantees that the technical efficient projection of $DMU_0$ that is provided, which is the point $(x_0^T, y_0^T) = (x_0 - s_0^{T-*}, y_0 + s_0^{T+*})$, where $s_0^{T-*}$ and $s_0^{T+*}$ denote an optimal solution of (2) and represent the slacks (i.e., the input excesses and the output shortfalls, respectively), actually belongs to the Pareto-efficient frontier of the PPS.

If profit ratios are used, then a measure technical efficiency can be defined as $TE_0 = \dfrac{\Pi_0}{\Pi_0^T}$, and the allocative efficiency measure is the corresponding residual $AE_0 = \dfrac{\Pi_0^T}{\Pi_0^*}$. Since (2) guarantees the achievement of the technical efficiency in the Pareto sense, $AE_0$ does not account for technical inefficiency and reflects the profit improvements that can be accomplished by substitutions along the efficient frontier. We have then the classical decomposition of the overall efficiency into its technical and allocative components: $OE_0 = TE_0 \times AE_0$. These three efficiency measures can be interpreted as the profit improvements achievable through each type of efficiency (overall, technical and allocative, respectively). They take values in (0,1] and satisfy the property of indication, which means that the value 1 of these measures characterizes the corresponding efficiency. They are also unit invariant and provide a unique decomposition of the overall efficiency that does not depend on the alternate optima of the models used.

We can proceed in a similar manner in the case of differences of profit, the technical efficiency measure being $TE'_0 = \dfrac{\Pi_0^T - \Pi_0}{p_0^t y_0 + c_0^t x_0}$ and the allocative one $AE'_0 = \dfrac{\Pi_0^* - \Pi_0^T}{p_0^t y_0 + c_0^t x_0}$, which give rise to an additive decomposition of the overall efficiency $OE'_0 = TE'_0 + AE'_0$. In this case, these three measures evaluate the normalized profits foregone through each type of inefficiency, so they have a similar interpretation to that of the measures based on profit ratios above defined and satisfy the same properties. The main difference is that now these are actually measures of inefficiency, and so, they do not so far take values in (0,1] but they are greater than or equal to 0. As a consequence, the value 0 is now that associated with the indication, i.e., $DMU_0$ is overall, technical or allocative efficient if, and only if, the value of the corresponding inefficiency measure equals 0.

## 3. Decomposing allocative profit efficiency into scale and mix effects

In this section we investigate how we can identify and measure both scale and mix effects in the allocative profit efficiency, within the context of the approach in Ruiz and Sirvent [8]. Following the reasoning in that approach, in order to determine how much of the profit improvement associated with the allocative efficiency can be explained by scale effects we have to determine the maximum profit that the technical efficient projection point (with profit $\Pi_0^T$) can achieve by modifying proportionally its inputs by a factor $\alpha$ and its outputs by a factor $\beta$. This profit, which is denoted by $\Pi_0^S$, is the optimal value of the following problem