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PHYSICA D
Physica D 211 (2005) 74–87
www.elsevier.com/locate/physd

Long tailed maps as a representation of mixed mode oscillatory systems

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Received 10 December 2004; received in revised form 19 July 2005; accepted 8 August 2005
Available online 9 September 2005
Communicated by U. Frisch

Abstract

Mixed mode oscillatory (MMO) systems are known to exhibit generic features such as the reversal of period doubling sequences and crossover to period adding sequences as bifurcation parameters are varied. In addition, they exhibit a nearly one dimensional unimodal Poincare map with a long tail. The numerical results of a map with a unique critical point (map-L) show that these dynamical features are reproduced. We show that a few generic conditions extracted from the map-L are adequate to explain the reversal of period doubling sequences and crossover to period adding sequences. We derive scaling relations that determine the parameter widths of the dominant windows of periodic orbits sandwiched between two successive states of RL^k sequence and verify the same with the map-L. As the conditions used to derive the scaling relations do not depend on the form of map, we suggest that the analysis is applicable to a family of two parameter one dimensional maps that satisfy these conditons.

PACS: 82.40Bj; 05.45Ac

Keywords: Multiple time scales; Mixed mode oscillations; One dimensional maps

1. Introduction

Dynamical systems with disparate time scales for the participating modes often exhibit periodic states characterized by a combination of relatively large amplitude and nearly harmonic small amplitude oscillations. Such periodic states are called the mixed mode oscillations (MMOs) conventionally denoted by L^s where L and s correspond to large and small amplitude oscillations, respectively. The associated complex bifurcation sequences consist of alternate periodic chaotic sequences which have been observed in models and experiments in the area of chemical kinetics [1–5], electrochemical reactions [6–8], biological systems [9], and in many physical systems [10–12]. These MMO systems typically exhibit the following features: (a) period doubling (PD) sequences and their reversal in multi-parameter space with respect to a primary

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0167-2789/$ – see front matter © 2005 Published by Elsevier B.V.
doi:10.1016/j.physd.2005.08.004

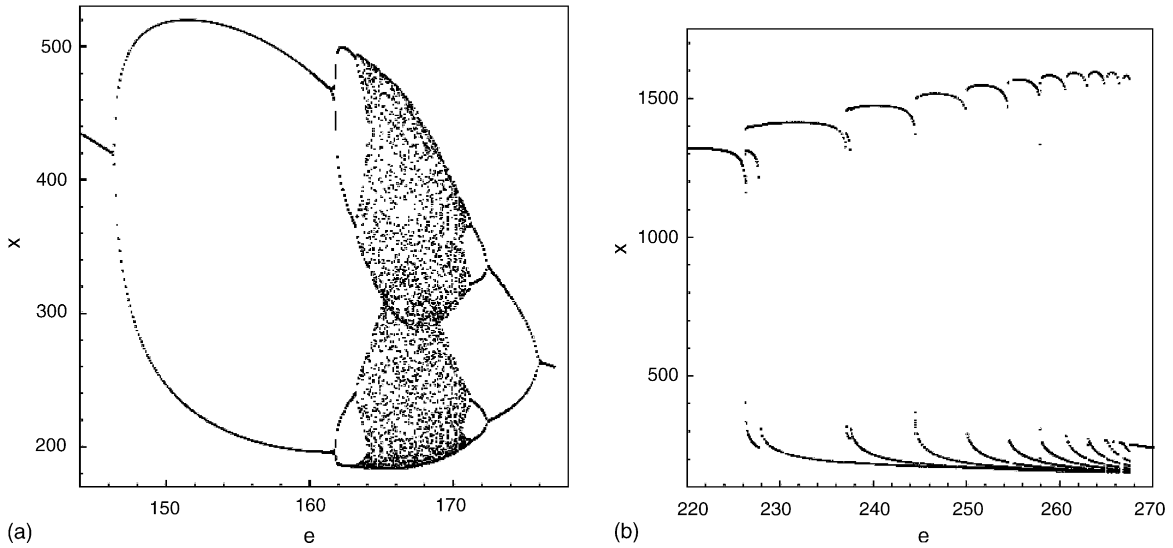

Fig. 1. Bifurcation diagrams for the AK model for a plastic instability with primary bifurcation parameter, e and secondary bifurcation parameter, m. (a) Cascading period doubling bifurcations and their reversals forming a bubble structure for $m = 2.16$, and (b) period adding sequences for $m = 1.2$. Chaotic regions exists in vanishingly small parameter regions sandwiched between successive 1^s periodic states.

bifurcation parameter keeping other parameters fixed, and (b) crossover to bifurcation sequences of alternate periodic-chaotic windows when one of the secondary bifurcation parameters is varied wherein dominant windows of periodicity increase in an arithmetical order which we refer to as period adding (PA) sequences [2,4,6–8,12,13]. Usually, MMO systems with these two features are also systems with multiple time scales of evolution.

As an illustration of the generic features of MMO systems, we collect some relevant results from our earlier study [12,14] on the Ananthakrishna's model (AK model) for a type of plastic instability [15]. As in other MMO systems, this model also involves disparate time scales. The bifurcation portraits (with respect to a primary bifurcation parameter, e) of the model show period doubling sequences and their reversal, gradually changing over to period adding sequences as the secondary bifurcation parameter, m, is decreased. (See Fig. 1a and b.) As can be seen in Fig. 1b, the dominant periodic orbits of 1^s kind constitute the period adding sequence. In the parameter space, wherein the reversal of period doubling sequences occurs, period adding sequences are finite. Periodic orbits of 1^s type lose their stability in a period doubling bifurcation as the (primary) control parameter is increased, restabilize through a reverse period doubling bifurcation, and are eventually annihilated in a fold bifurcation [12]. We have also studied the next maximal amplitude (NMA) maps [16] obtained by plotting one maximum of the evolution of the fast variable with the next [14]. (The NMA maps can be regarded as a specific form of the Poincare maps.) These NMA maps show a near one dimensional unimodal nature with features of sharp maximum (i.e., the ratio of the height to the width being large)[1] and with a long tail as seen in the maps shown in Fig. 2a and b. (Also, see [14,17].) Indeed, several other MMO systems also exhibit features stated above (and also displayed in Figs. 1 and 2) [5,13,18].

A well studied example of MMO systems is the Belusov Zhabotinsky (BZ) reaction system. Exhaustive theoretical/experimental studies for the BZ systems in two parameter space have shown that the NMA maps of these systems have a unimodal structure with a long tail and show a similar trend as a function of the control parameters [5,13,19] as in the case of AK model. Other

[1] For a one dimensional unimodal map, an acceptable definition would be $p - f^{-1}(p)$ is much less than $f(c) - p$ where $f^{-1}(p)$ is the only one preimage of p distinct from itself. Refer Fig. 3 for notations.