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Surfactant-induced fingering phenomena in thin film flow down an inclined plane

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Abstract

We consider the dynamics of a thin liquid film covered with insoluble surfactant flowing down an inclined plane. A coupled pair of nonlinear partial differential equations for the film height and surfactant concentration are derived using lubrication theory. Two configurations of fluid and surfactant are considered: constant flux and constant volume. Examination of the base states reveals the presence of propagating wave fronts in both configurations. Application of a transient growth analysis demonstrates the existence of an instability near the leading edge of the fluid that is found to be vulnerable to transverse disturbances of intermediate wavenumber. Our results illustrate that several features of the much studied uncontaminated film flow problem are modified due to the inclusion of surfactant.

Keywords: Thin films; Instability; Finger formation; Surfactants; Marangoni effects; Inclined flow

1. Introduction

Thin liquid films are central to numerous engineering, materials processing, and biomedical applications [1]. In situations involving gravitationally-driven coating flows, an advancing fluid front is susceptible to a transverse instability that can lead to rivulet formation [2]. This instability is detrimental to many industrial processes such as spin coating, for instance see [3]. These experimental observations have been complimented by modelling studies that isolated the mechanisms responsible for fingering at the driven fluid front, [4–7]. Further experimental work has produced detailed images of fingering patterns formed for various angles of inclination [8], while more recent work

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has focused on modelling of the instability in the nonlinear regime [9–11], with methods proposed for controlling rivulet formation [12,13].

The studies mentioned above have focused on essentially uncontaminated liquid films. In the presence of surfactant, gradients in surfactant concentration give rise to Marangoni stresses that will effect the film dynamics. These surfactant-driven flows have been subject to many studies in connection with surfactant replacement therapy (SRT) [14–16], a treatment for both neonate and adult respiratory distress syndrome, and levelling of thin coating layers [17]. The spreading of surfactant-laden films is also accompanied by another transverse instability (see [18–25] and references therein). Note that in this case, finger formation occurs on horizontal substrates, that is, in the absence of gravitational forcing.

In this paper, we investigate the dynamics of a gravitationally-driven thin film partially covered with an insoluble surfactant present in dilute concentrations. Evolution equations for the film thickness and surfactant concentration are derived in the lubrication approximation. Constant flux [26] and constant volume [27,28] configurations are studied with a precursor layer used to relieve the singularity at the contact line. The model incorporates the effects of gravity, Marangoni stresses, inclination angle, precursor layer, capillarity, and surface diffusion on the flow, while intermolecular forces are neglected. The stability of the flow is examined using a transient growth analysis. Our results indicate that the presence of surfactant significantly modifies the flow characteristics of a gravitationally-driven thin uncontaminated film.

2. The model

2.1. Evolution equations

A thin film of incompressible, Newtonian fluid of constant viscosity μ, and density ρ, rests on an inclined plane bounded from above by an inviscid gas and below by a rigid, impermeable solid substrate. The advancing film is partially covered with an insoluble surfactant of initially uniform concentration, Γ_m, which is assumed to be much smaller that the saturation concentration. Two different flow configurations are considered: constant flux and constant volume (see Fig. 1). In the former case, the film is connected to a reservoir at the flow origin providing a constant flux of fluid and surfactant. In the latter, a droplet of constant volume initially rests atop a precursor layer of uniform thickness, H_b [4]. The liquid–air interface is located at $z = h(x, y, t)$, where x, y and z denote the streamwise, transverse and vertical coordinates, respectively, while t denotes time. The film aspect ratio $\epsilon = H/L$ is taken to be small; here H is the characteristic depth of the film and L its characteristic lateral extent.

The evolution equations for the film thickness and surfactant concentration have been derived elsewhere [26]. Thus we provide only the relevant scalings and the final evolution equations which will be studied. The scalings adopted are given by: $x = L\bar{x}$, $y = L\bar{y}$, $z = H\bar{z}$, $u = U\bar{u}$, $v = V\bar{v}$, $w = \epsilon U\bar{w}$, $t = (\mu L^2/\Pi H)\bar{t}$, $p = (\Pi/H)\bar{p}$, $\bar{\sigma} = (\sigma - \sigma_m)/\Pi$, $\Gamma = \Gamma_m\bar{\Gamma}$, where u, v, w, p, denote the streamwise, transverse, normal components of the velocity field and pressure, respectively; overbars denote dimensionless quantities. Here, $\Pi = \sigma_o - \sigma_m$, the spreading pressure, is the surface tension difference between the maximal and minimal value of the surface tension, denoted by σ_o and σ_m, respectively. Note also that $U = \Pi H/\mu L$ is the characteristic Marangoni velocity. The overbars are henceforth suppressed.

The two-dimensional non-linear evolution equations for h and Γ are given by,

$$h_t = \nabla \cdot \left[\frac{h^3}{3}(-C\nabla\nabla^2 h) + \frac{h^2}{2}\nabla\Gamma\right] - \left[\frac{h^3}{3}G\sin\theta\right]_x, \tag{1}$$

$$\Gamma_t = \frac{\nabla^2\Gamma}{Pe} + \nabla \cdot \left[-\frac{h^2}{2}\Gamma C\nabla\nabla^2 h + h\Gamma\nabla\Gamma\right] - \left[\frac{h^2}{2}\Gamma G\sin\theta\right]_x. \tag{2}$$