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Defect structures in sine-Gordon like models

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Abstract

We investigate several models described by real scalar fields, searching for topological defects. Some models are described by a single field, and support one or two topological sectors, and others are two-field models, which support several topological sectors. Almost all the defect structures that we find are stable and finite energy solutions of first-order differential equations that solve the corresponding equations of motion. In particular, for the double sine-Gordon model we show how to find small and large BPS solutions as deformations of the BPS solution of the ϕ^4 model. And also, for most of the two field models we find the corresponding integrating factors, which lead to the complete set of BPS solutions, nicely unveiling how they bifurcate among the several topological sectors.

Keywords: Defect structures; Kinks; Domain walls

1. Introduction

This work deals with kinks and domain walls in sine-Gordon like models in (1,1) spacetime dimensions. They are extended classical solutions of topological profile, supposed to play important role in several different contexts as for instance in condensed matter [1,2] and in high energy physics [3,4].

Models described by real scalar fields in (1,1) space-time dimensions are among the simplest systems that support topological solutions. Usually, the topological solutions are classical static solutions of the equations of motion, with topological behavior related to the asymptotic form of the field configurations. The topological profile can be made quantitative, with the inclusion of the topological current, $j^\mu_{\rm T} = (1/2)\varepsilon^{\mu\nu}\partial_\nu\phi$. It gives the topological charge $Q_{\rm T} = (1/2)(\phi(x\to\infty) - \phi(x\to-\infty))$, which is not zero when the asymptotic value of the field differs in both the positive and negative directions. To ensure that the classical solutions have finite energy, one requires that

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the asymptotic behavior of the solutions is identified with minima of the potential that defines the system under consideration, so in general the potential has to include at least two distinct minima in order for the system to support topological solutions.

We can investigate real scalar fields in (3,1) space-time dimensions, and now the topological solutions are named domain walls. These domain walls are bidimensional structures that carry surface tension, which is identified with the energy of the classical solutions that spring in (1, 1) space-time dimensions. The domain wall structures are supposed to play a role in applications to several different contexts, ranging from the low energy scale of condensed matter [1,2,5,6] up to the high energy scale required in the physics of elementary particles, fields and cosmology [3,7,4].

There are at least three classes of models that support kinks or domain walls, and we further explore such models in the next Section 2. In the first class of models one deals with a single real scalar field, and the topological solutions are structureless. Examples of this are the sine-Gordon and ϕ^4 models [3]. In the second class of models one also deals with a single real scalar field, but now the systems comprise at least two distinct domain walls. An example of this is the double sine-Gordon model, which has been investigated for instance in Refs. [8–13]. In the third class of models we deal with systems defined by two real scalar fields, where one finds domain walls that admit internal structure [14–20], and junctions of domain walls, which appear in models of two fields when the potential contains non-collinear minima, as recently investigated for instance in refs. [21–33]. We study new possibilities in Section 3, and there we investigate periodic systems, which present several topological sectors, which may bifurcate into richer structures. For most of them, we find the integrating factors, which lead us to all the BPS solutions the models engender.

There are other motivations to investigate domain walls in models of field theory, one of them being related to the fact that the low energy world volume dynamics of branes in string and M theory may be described by standard models in field theory [34–36]. Besides, one knows that field theory models of scalar fields may also be used to investigate properties of quasi-linear polymeric chains, as for instance in the applications of refs. [37–40], to describe solitary waves in ferroelectric crystals, the presence of twistons in polyethylene, and solitons in Langmuir films.

Domain walls have been observed in several different scenarios in condensed matter, for instance in ferroelectric crystals [6], in one-dimensional nonlinear lattices [41], and more recently in higher spatial dimensions—see [42] and references therein. The potentials that appear in the models of field theory that we investigate in the present work are also of interest to map systems described by the Ginzburg–Landau equation, since they may be used to explore the presence of fronts and interfaces that directly contribute to pattern formation in reaction-diffusion and in other spatially extended, periodically forced systems [5,2,43–46].

For completeness, in Section 4 we investigate other topological sectors of the periodic two field models, where the domain wall solutions are of the non-BPS type. And finally, we end the paper in Section 5, where we include our comments and conclusions.

2. Single field models

In this work we are interested in field theory models that describe real scalar fields and support topological solutions of the Bogomol'nyi–Prasad–Sommerfield (BPS) type [47,48]. In the case of a single real scalar field ϕ, we consider the Lagrange density

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) \tag{1}$$