Available online at www.sciencedirect.com
SCIENCE DIRECT®
Physica D 207 (2005) 41–51
PHYSICA D
www.elsevier.com/locate/physd

Nonlinear dynamics of traffic jams

Tong Li*

Department of Mathematics, University of Iowa, 14 MacLean Hall, Iowa City, IA 52242, USA

Received 2 September 2004; received in revised form 11 May 2005; accepted 13 May 2005
Available online 6 June 2005
Communicated by J. Lega

Abstract

A class of traffic flow models that capture the nonlinear dynamics of traffic jams are proposed. The class of discrete models originate from equations satisfied by the discrete traveling waves of some inviscid nonequilibrium continuum models. The self-organized oscillatory behavior and chaotic behavior in traffic are identified and formulated. The results can help to explain the appearance of a phantom traffic jam observed in real traffic flow.

There is a qualitative agreement when the analytical results are compared with the empirical findings for freeway traffic and with the previous numerical simulations.
© 2005 Elsevier B.V. All rights reserved.

AMS Classification: 35B30; 35B40; 35L65; 76L05; 90B20

Keywords: Traffic jams; Chaos; Stop-and-go waves; Bifurcation; Discrete traveling waves; Nonconcavity

1. Introduction

Experimental observations of real traffic have revealed rich nonlinear phenomena: the formation of traffic jams, stop-and-go waves, hysteresis and phase transitions, see Daganzo et al. [9], Helbing [19], Kerner [24,25], Kerner and Rehborn [27], Knospe, Santen, Schadschneider and Schreckenberg [29], Mauch and Cassidy [42], Treiterer and Myers [52]. Traffic systems exhibit extremely complex behavior derived from several sources: the heterogeneous nature of human behavior, highly nonlinear group dynamics and large system dimensions. Many modelling approaches have been suggested by traffic engineers, physicists and mathematicians. They use either discrete or continuous state space, with discrete or continuous time and/or space, see Bando et al. [2], Bellomo, Delitala and Coscia [3], Berg and Woods [4], Colombo and Groli [7], Gazis, Herman, and Rothery [11], Gray and Griffeath [12], Greenberg, Klar and Rascle [15], Günther, Klar, Materne and Wegener [17], Hayakawa and Nakanishi [18], Helbing [20], Hermann and Kerner [21], Illner, Klar and Materne [22], Jin and Zhang [23], Kerner and Konhäuser [26], Kühne [30], Li [36], Lighthill and Whitham [40], Nagel [43], Payne [45], Prigogine and Herman [46], Richards [47], Smilowitz and Daganzo [50], Zhang [55]. Theoretical

* Tel.: +1 3193343342; fax: +1 3193350627.
E-mail address: tli@math.uiowa.edu.

0167-2789/$ – see front matter © 2005 Elsevier B.V. All rights reserved.
doi:10.1016/j.physd.2005.05.011

modelling allows simulations of large traffic networks and opens perspectives for future applications such as traffic forecasts and dynamic route guidance systems.

Intrigued by the phenomenology of real-world traffic and the simulations of others, we propose an innovative approach to the nonlinear dynamics of traffic flow. The class of discrete models are derived from nonequilibrium continuum models. We are able to identify and predict the self-organized oscillatory behavior. Comparing our analytical results with the empirical findings for freeway traffic, we see a qualitative agreement.

Previously, Helbing [19] reviewed results on empirical findings and modeling approaches of congested traffic. Gray and Griffeath [12] studied the ergodic theory of traffic jams. They analyzed a probabilistic cellular automaton (CA) as a prototype for the emergence of traffic jams at some intermediate density range, also see Nagel [43]. Schadschneider and Schreckenberg [49] obtained stop-and-go waves from CA models.

Kerner and Konhäuser [26], Jin and Zhang [23] obtained clustering solutions when they numerically solved the viscous and inviscid Payne-Whitham Eqs. (9) and (10) respectively, with nonconcave fundamental diagram (16) in *the unstable regions*. Clusters in traffic flow are characterized by the appearance of a region of high density and low average velocity of vehicles in an initially homogeneous flow. In the stable region (12) where the sub-characteristic condition is satisfied, Li and Liu [38] showed that the traveling wave solutions of the inviscid Payne-Whitham equations with nonconcave fundamental diagrams are asymptotically stable under small disturbances and under the sub-characteristic condition. In their numerical simulations of the viscous Payne-Whitham Eqs. (9) and (10) with a nonconcave fundamental diagram (18), Lee et al. [33] triggered a form of stop-and-go traffic. Greenberg, Klar and Rascle [15] found wave train solutions for a model for traffic on a multilane freeway. Bando et al. [2] proposed an optimal velocity model and observed the evolution of traffic congestion. Gasser, Sirito and Werner [10] performed bifurcation analysis of a class of car-following traffic models. Numerical simulations of Greenberg [14] yielded the periodic traveling wave solutions for a higher-order traffic flow model on a ring road with fundamental diagram (19). In all the continuum models, nonconcave fundamental diagrams were used to obtain the oscillatory solutions. A key physical condition in obtaining clustering solutions in continuum models is that the fundamental diagrams change concavity.

The paper is organized as follows. We present important approaches toward the modeling of traffic phenomena in Section 2. We derive the discrete models in Section 3. In Section 4, we show existence of clustering solutions and compare them with the previous observations and numerical simulations. We conclude in Section 5.

2. Preliminaries

There are various important approaches to the modeling of traffic phenomena: microscopic models which explain traffic phenomena on the basis of the behavior of single vehicles [11], mesoscopic models such as kinetic or Boltzmann-like models [19,28,46], and macroscopic models which describe traffic phenomena through parameters which characterize collective traffic properties [1,8,13,35,36,39,40,45,53,55].

Assuming that there exists a function relation between the velocity and the density $v = v_e(\rho)$, Lighthill and Whitham [40] and Richards [47] developed the first macroscopic model of traffic flow, LWR (Lighthill, Whitham and Richards) theory,

$$\rho_t + (\rho v_e(\rho))_x = 0. \quad (1)$$

The initial data is

$$\rho(x, 0) = \rho_0(x) > 0. \quad (2)$$

$v_e(\rho)$ is a non-increasing function

$$v_e'(\rho) \leq 0. \quad (3)$$

$v_e(0) = v_f$ and $v_e(\rho_j) = 0$, where v_f is the free flow speed and ρ_j is the jam concentration.

$$q(\rho) = \rho v_e(\rho) \quad (4)$$

is the so-called fundamental diagram in traffic flow. There are a huge number of different suggestions about the speed-density relation. The fundamental diagrams may be concave, see Greenshields [16], nonconcave, smooth, discontinuous or have multiple branches. We focus our attention on traffic flow models with single-valued fundamental diagrams that change concavity. It

Download English Version:

https://daneshyari.com/en/article/9877601

Download Persian Version:

https://daneshyari.com/article/9877601

Daneshyari.com