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Complicated dynamics of a linear oscillator with a light, essentially nonlinear attachment

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Abstract

We study the dynamics of a two-degree-of-freedom (DOF) nonlinear system consisting of a grounded linear oscillator coupled to a light mass by means of an essentially nonlinear (nonlinearizable) stiffness. We consider first the undamped system and perform a numerical study based on non-smooth transformations to determine its periodic solutions in a frequency–energy plot. It is found that there is a sequence of periodic solutions bifurcating or emanating from the main backbone curve of the plot. We then study analytically the periodic orbits of the undamped system using a complexification/averaging technique in order to determine the frequency contents of the various branches of solutions, and to understand the types of oscillation performed by the system at the different regimes of the motion. The transient responses of the weakly damped system are then examined, and numerical wavelet transforms are used to study the time evolutions of their harmonic components. We show that the structure of periodic orbits of the undamped system greatly influences the damped dynamics, as it causes complicated transitions between modes in the damped transient motion. In addition, there is the possibility of strong passive energy transfer (energy pumping) from the linear oscillator to the nonlinear attachment if certain periodic orbits of the undamped dynamics are excited by the initial conditions.

Keywords: Resonance capture; Essential nonlinearity; Energy transfer

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1. Introduction

The dynamics of multi-degree-of-freedom (DOF) coupled oscillators with or without forcing has been investigated in numerous previous works. Complicated dynamics caused by internal, external or combination resonances has been studied, such as bifurcations of equilibrium positions and periodic or quasiperiodic orbits, nonlinear instabilities, self-excited oscillations, sudden transitions between co-existing stable solutions, transitions to chaos or 'chaotic explosions', complex broadband energy transfers between system components, resonance captures, and increases in the dimensionalities of attractors with varying parameters [1–4]. In this work, we study the dynamics of a two-DOF nonlinear system with surprisingly complicated dynamics. It consists of a grounded linear oscillator coupled to a light mass by means of an essentially nonlinear (nonlinearizable) stiffness. As shown in previous works [5,6], in this class of coupled oscillators interesting energy exchange phenomena can occur, including one-way irreversible transfer of energy from the linear oscillator to the nonlinear attachment (a phenomenon termed *nonlinear energy pumping* in these works). Such energy exchanges are often associated with *resonance captures* [7,8], whereby the nonlinearizable, nonlinear oscillator engages in transient resonance with the linear oscillator, before the dynamics 'escape' to a different regime of the motion.

Up to now, only grounded and relatively heavy nonlinear attachments were considered, a feature that limits their applicability to practical applications. To eliminate these restrictions, an ungrounded and light nonlinear attachment is considered herein, which, in addition, possesses the feature of simplicity. Even though the system considered in this work has a simple configuration, still, it possesses a very complicated structure of undamped periodic orbits, which, in turn, gives rise to complicated series of transitions and energy exchange phenomena in the damped dynamics.

The structure of the paper is as follows. In Section 2, we perform a numerical study based on the method of non-smooth transformations first developed by Pilipchuk [9,10] in order to study the various branches of periodic solutions in a frequency–energy plot; it is found that this system possesses a sequence of periodic solutions bifurcating or emanating from the main backbone of the frequency–energy plot. In Section 3, the periodic orbits of the undamped system are studied analytically using the complexification/averaging technique introduced by Manevitch [11], which enables the systematic treatment of the frequency contents of the various branches of solutions, and the understanding of the type of oscillations performed by the system at different regimes of the motion. Finally, in Section 4, the transient responses of the weakly damped system are examined, and numerical wavelet transforms are used to study the time evolutions of their harmonic components; the results show that the structure of periodic orbits of the undamped system greatly influences the damped dynamics, as it causes complicated transitions between modes in the damped transient motion. In addition, it is shown that strong passive energy transfer from the directly excited linear oscillator to the nonlinear attachment is possible by exciting certain periodic orbits of the undamped dynamics.

2. Numerical study of the periodic orbits of the undamped system

The system considered is depicted in Fig. 1. It consists of a linear oscillator of mass m_1 (the 'linear oscillator') that is coupled through an essentially nonlinear stiffness to a mass m_2 (the 'nonlinear attachment'). The equations of motion of this two-DOF system are given by:

$$\begin{aligned} &m_1\ddot{x} + k_1 x + c_1\dot{x} + c_2(\dot{x}-\dot{v}) + k_2(x-v)^3 = 0 \Rightarrow \ddot{x} + \omega_0^2 x + \lambda_1\dot{x} + \lambda_2(\dot{x}-\dot{v}) + C(x-v)^3 = 0,\\ &m_2\ddot{v} + c_2(\dot{v}-\dot{x}) + k_2(v-x)^3 = 0 \Rightarrow \varepsilon\ddot{v} + \lambda_2(\dot{v}-\dot{x}) + C(v-x)^3 = 0, \end{aligned} \tag{1}$$

where $\omega_0^2 = k_1/m_1$, $C = k_2/m_1$, $\varepsilon = m_2/m_1$, $\lambda_1 = c_1/m_1$, and $\lambda_2 = c_2/m_1$. The motivation for examining this system is to study in detail energy transfer exchanges occurring between the two degrees of freedom. More specifically, considering direct impulsive forcing of only the linear oscillator, we are interested to examine the possibility