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A comparison of one-dimensional traveling waves in inverse and normal fluidized beds

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Abstract

The state of uniform fluidization is usually unstable to small disturbances, and this can lead to the formation of vertically traveling voidage waves. In inverse fluidization, when particle density is less than fluid density ($\rho_s < \rho_f$), particles fluidize in the direction of gravity when the drag force exerted by the fluid overcomes buoyancy. Inverse fluidization thus provides a unique parameter space, which augments the study of instability behavior in normal fluidization when $\rho_f < \rho_s$. Using continuum equations of continuity and motion, we compared the linear stability of normal and inverse bed modes to examine the effect of the Froude number (*Fr*) and fluid to solid density ratio ($\delta = \rho_f/\rho_s$). Making use of numerical bifurcation analysis and continuation, periodic solutions in the form of one-dimensional traveling waves (1D-TWs) were computed. Based on wave growth rates and bifurcation structure, we identified the *Fr* as an important parameter for predicting instability *strength*. However, δ affects instability *onset*, or the point at which the base state is rendered unstable. In the case studies we examined, traveling waves were shown to propagate in the direction of fluidization, and asymmetrical, high amplitude 1D-TW profiles suggest fully developed bubble-like structures are oriented in the direction of fluidization.

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1. Introduction

In inverse fluidization, low density particles become mobile, or *fluidize*, when the drag exerted by a heavier fluid flowing downwards through the column overcomes the buoyancy force on the particles [1]. Inverse fluidization is

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the reverse of what is considered to be normal fluidization, where heavier particles are fluidized by the upwards flow of a lighter gas or liquid. Fluidizing lightweight particles by a heaver medium is advantageous in many important industrial applications where enhanced multi-phase mixing can improve heat and mass transfer performance (see [2]). For example, in biotechnology and catalytic chemical reaction engineering, inverse turbulent three-phase reaction systems have been investigated for improved selectivity and yield. In these systems, lightweight particles are fluidized by the countercurrent flow of liquid downwards and gas bubbles upwards [3–5]. In fluidized-bed dry particle coating, a high-density super critical fluidization medium (operating in inverse mode) may improve coating efficiency by affecting the frequency and impact value of particle–particle collisions. However, it is difficult to support the use of this mode as a viable alternative without a better understanding of how fluidization *direction* (relative to gravity) affects instability behavior in the bed.

In normal fluidized beds, it has been well-documented that the base-state of uniform fluidization is usually unstable to small disturbances, and this can lead to the formation and propagation of vertically traveling *voidage waves*. When primary instabilities becomes spatially amplified in the bed, this can bring about complex bubbling and turbulent flow regimes, which completely alter the flow characteristics of the system [6]. In gas–fluidized beds, voidage waves are in the form of *bubbles*, where experimental evidence has shown that just beyond conditions of minimum fluidization, the solids tend to remain compacted as increasing volumes of gas pass through the condensed phase “much in the manner of a gas passing through an actual liquid” [7]. This mode of fluidization is often referred to as *aggregative*, and differs dramatically from flow behavior that is sometimes observed in liquid–fluidized beds, which expand uniformally and are generally more stable in operation (referred to as non-bubbling or *particulate*).

In the fluidization research, *two-phase* continuum models have been used to study the stability behavior of gas- and liquid–fluidized beds. This approach uses *ensemble*- or volume-averaged equations of continuity and motion to describe the behavior of the fluid and particle phases, and has been demonstrated repeatedly to capture the physics necessary to distinguish between bubbling and non-bubbling systems (see [8] for discussion). This approach uses constitutive relationships or closure laws to express the various force terms as functions of locally averaged variables. Researchers have generally adopted closures based on empirical correlations [9–12], but constitutive terms have also been theoretically derived using physical arguments [13], and from first principles [14]. Anderson et al. [15] successfully demonstrated that these equations do capture the physics necessary to distinguish between bubbling and non-bubbling systems. Recently, Duru et al. [16] tested this approach experimentally by relating the physical properties of saturated voidage waves to the particle phase pressure and viscosity terms. Their results confirmed that the model was satisfactory for describing the behavior of one-dimensional voidage waves within the experimental parameter range investigated (see also [17]).

In the experimental work of Wilhelm and Kwauk [7], solid–air (or aggregative) systems were found to be separable from solid–water (or particulate) systems on the basis of the dimensionless Froude number evaluated at minimum fluidization velocity, for a wide range of particle species. Experimental evidence of such distinct flow behavior has prompted its investigation by linear stability analysis of the uniform fluidization state. In a stability analysis of gas- and liquid–fluidized beds, Göz [18,19] analyzed primary bifurcations of two-dimensional vertically and oblique traveling waves from the base-state, and found only minor differences between gas- and liquid–fluidized beds. Göz [20] also found similar bifurcation structure exhibited in gas- and liquid–fluidized beds having small *Fr* approximations. Göz and Sundaresan [21] extended a previous analysis performed by Göz [22], to examine the stability of one-dimensional periodic waves to two-dimensional perturbations of large transverse wavelength in liquid–fluidized beds by considering the effects of fluid phase inertia and viscosity. These authors demonstrated that the instability mechanism is the same for both gas- and liquid–fluidized beds, and concluded that scaling differences play an important role in distinguishing the difference in gas- and liquid–fluidized bed behavior, viz. the *Fr* number group.

Linear stability analyses of the base state have since led to the computation of fully-developed, one and two-dimensional traveling wave solutions using numerical simulation techniques and bifurcation theory [23]. These authors found that for both gas- and liquid–fluidized beds, two-dimensional traveling waves were subsequently born out of one dimensional traveling wave solutions emerging through Hopf bifurcations of the steady state