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Dynamics of defects and traveling waves in an interfacial finger pattern

Piotr Habdas[a], John R. de Bruyn[b,c,*]

[a] *Department of Physics, St. Joseph's University, Philadelphia, PA 19131, USA*
[b] *Department of Physics and Physical Oceanography, Memorial University of Newfoundland, St. John's, Nfld, Canada A1B 3X7*
[c] *Laboratoire de Rhéologie, B.P. 53, Domaine Universitaire, Grenoble 38041 Cedex 9, France*

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Abstract

We present the results of an experimental study of one-dimensional traveling finger patterns dominated by source and sink defects. The system studied is a driven fluid–air interface, and the traveling-wave state arises via a subcritical bifurcation. We analyze space–time images of the patterns to obtain the amplitude and local wave number of the left- and right-going traveling waves near the defects. We determine the width of the defects, and find that the width of the sinks does not vary significantly with distance above the bifurcation over the range of our experiments, while the source width increases as the bifurcation is approached from above. We also observe both localized depressions or enhancements in wave amplitude and periodic modulations of the pattern amplitude and wave number.

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1. Introduction

Source and sink defects have been observed in a variety of one-dimensional pattern-forming systems with traveling-wave states [1–10]. These defects separate regions of oppositely moving traveling waves: sources emit left- and right-going waves, while sinks absorb them. They are the fundamental defects of one-dimensional traveling-wave patterns, and an understanding of their behavior is key to understanding the dynamics of more complex spatio-temporal systems.

Sources and sinks have been studied theoretically in the context of coupled complex Ginzburg–Landau equations (CCGLE) [11]. The properties of sources and sinks in travelling-wave patterns which develop above a supercritical Hopf bifurcation have been analyzed by several groups [12–17]. van Hecke et al. carried out a

* Corresponding author.
E-mail address: jdebruyn@physics.mun.ca (J.R. de Bruyn).

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detailed study of defects in solutions to CCGLE [16]. Their simulations indicated that sources control the dynamics of the pattern. Sources were found to be stable and symmetric with respect to the wave number of the pattern emitted on either side of the source above a certain critical value of the control parameter ε. Below this value of the control parameter sources become unstable and non-stationary [12] and start to “breathe”, that is, their width varies with time. This transition in defect behavior is associated with the transition from convective to absolute instability of the travelling-wave state [18–21]. Sinks, on the other hand, are essentially passive. The wave number of the pattern on the two sides of a sink can be different, and their behavior did not change as a function of ε. The widths of both sources and sinks were found to diverge as ε^{-1} close to the supercritical Hopf bifurcation at $\varepsilon = 0$ [16]. Coherent structures in the wave pattern have also been studied theoretically. So-called holes in the travelling-wave pattern – that is, localized depressions in the amplitude of the wave, coupled with a change in the wave number – have been observed in solutions to CCGLE [17,22–24]. Nozaki–Bekki holes [22] connect regions of traveling waves with different wave number, while homoclinic holes or homoclons [17,23,24] connect regions with the same wave number. Modulated amplitude waves, which are coherent traveling modulations in both amplitude and wave number, have been studied recently [17,25–27]. Although both holes and modulated amplitude waves tend to be linearly unstable, they nonetheless play an important role in the development of spatio-temporal chaos and complex dynamics in solutions of the CCGLE.

Experimentally, sources and sinks have been studied extensively in traveling-wave convection driven by a heated wire [1–3,21], as well as in other systems [4–9]. Traveling waves in heated-wire convection appear via a supercritical Hopf bifurcation, as in the CCGLE. Alvarez et al. [2] found sources to be stationary and symmetric – that is, the wave number emitted was the same on each side of the defect – but different sources emitted waves of different wave number. They also found that sinks sandwiched between two patches with different wave numbers moved such that the phase difference across the source remained fixed. This phase-matching behavior does not appear in solutions to the CCGLE, which do not treat the short length scales involved. The experiments of Garnier et al. [21] showed a logarithmic variation in source width which was consistent with predictions for the behavior of defects near the convective-absolute instability transition [19]. Pastur et al. [3] found that below a critical value of the control parameter ε, sources started to fluctuate strongly and their width diverged as ε^{-1}. Sinks moved according to the above phase-matching rule, and their width did not diverge as ε was decreased. They observed homoclinic holes in the pattern which were emitted from sources and propagated through the pattern, and observed holes connecting regions of different wave number in transient patterns when the driving force was changed. Additional experimental evidence for both holes [28–30] and modulated amplitude waves [31–34] in other traveling-wave systems has recently been reviewed by van Hecke [17].

The printer’s instability (also called directional viscous fingering) [10] is a system well suited to the study of the dynamics of one-dimensional patterns in extended systems [35]. The system consists of a fluid–air interface confined in the gap between two acentrically mounted horizontal cylinders. When the system is driven away from equilibrium by rotating the cylinders, a variety of fingering patterns can develop on the interface. Among the phenomena which have been studied in this system are stationary finger patterns [10,36], traveling finger patterns [8,10,37–39] which arise from a parity-breaking instability [37,38,40,41], spatio-temporally chaotic patterns [8,10,42], and sink and source defects [8–10] which separate regions of oppositely traveling fingers. Habdas et al. [9] studied the behavior of sink and source defects in this system. They found that sinks separate regions of differing wave number and move with the phase-matching behavior described above. Sinks were observed to be transient objects which eventually annihilated by collision with a source or the boundaries of the system. Isolated sources, on the other hand, persisted for the duration of the experiment. They were symmetric and stationary on average, although individual sources underwent small-scale random motion apparently driven by noise.

In this paper we describe the results of a study of the amplitude and local wave number near source and sink defects in the printer’s instability. Using Fourier decomposition, we analyze the amplitudes of the counterpropagating waves on each side of the defects. From this analysis we determine the width of the defects as