Structural Change and Economic Dynamics 33 (2015) 71–85

Contents lists available at ScienceDirect

Structural Change and Economic Dynamics

journal homepage: www.elsevier.com/locate/sced

A geometrical approach to structural change modelling

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ARTICLE INFO

Article history:
Received June 2014
Received in revised form February 2015
Accepted April 2015
Available online 17 April 2015

JEL classification:
C61
C65
O41

Keywords:
Structure
Dynamics
Geometrical
Simplex
Path dependency

ABSTRACT

A large body of research has studied structural change, particularly the dynamics of labour allocation in multi-sector growth models, based on different economic theories. We present a meta-model of structural change that combines information from empirical evidence (stylized facts) with information on the geometrical properties of typical trajectories studied in structural change theory. In particular, our approach is based on three facts: (1) structural change in three-sector models is defined on a 2-simplex; (2) the trajectory of past structural change partitions the 2-simplex into economically interpretable sections; (3) the typical properties of structural change trajectories (e.g. non-self-intersection) prohibit some movements from one section to another. Jointly, these facts imply that structural change is path-dependent and that the number of feasible structural change scenarios can be reduced significantly. While we focus on labour allocation dynamics, our approach can be applied to other topics (e.g. income distribution dynamics).

1. Introduction

A large body of research[1] has studied structural change, particularly the dynamics of labour allocation in multi-sector growth models, based on different economic theories. The meta-model of structural change presented in this paper combines information from empirical evidence (stylized facts) with information on the geometrical properties of typical trajectories studied in structural change theory.[2] We show that structural change is path-dependent in our model and use this fact to reduce the number of future structural change scenarios significantly.

Most of the quantitative and qualitative methods in economic dynamics (e.g. vector auto regression, simulation and phase-diagram analysis) combine theoretical and empirical information on the subject of analysis to derive some predictions regarding future dynamics (or policy responses). We do the same. However, in contrast to the majority of quantitative and/or predominantly theoretical approaches (in structural change theory), we do not follow specific economic doctrines, but use the mathematical assumptions common to most structural change models. This idea results in a qualitative meta-model of structural change.

Our approach can be summarized as follows. First, we show that the dynamics of a three-sector economy (agriculture, manufacturing, services) can be modelled on a 2-simplex[3]; in other words, the 2-simplex is the domain of the structural change trajectory. Second, we provide an

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[1] Some recent contributions are, for example, Kongsamut et al. (2001), Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008), Foellmi and Zweimueller (2008) and Buera and Kaboski (2009). See Section 4 for survey papers.

[2] Our approach is similar to the "qualitative simulation algorithms" used in physics; see, for example, Kuipers (1986) and Lee and Kuipers (1988).

[3] Works that use standard simplexes for economic modelling include Neumann and Morgenstern (1953), McKenzie (1955), Border (1985), Leamer (1987), and Weibull (1995).

http://dx.doi.org/10.1016/j.strueco.2015.04.001
0954-349X/

economic interpretation of the points and trajectories on the 2-simplex. Third, we collect information from widely accepted stylized facts on structural change and translate it into dynamics on the 2-simplex. Fourth, the structural change trajectories that arise in structural change theory have some specific geometrical properties; we use the fact that they are continuous and do not intersect themselves (on the 2-simplex). Fifth, by combining the information derived from the previous steps, we obtain a qualitative meta-model of structural change that implies that today's structural change depends on past structural change (i.e. structural change is path-dependent). Finally, we show that this path dependency restricts the set of feasible structural change scenarios significantly (for a summary of these scenarios, see Section 9). This fact can be used in structural change predictions.

The remainder of the paper is organized as follows. We show in Section 2 that structural change in the three-sector model can be depicted on a 2-simplex. In Section 3, we elaborate the interpretation of the points and trajectories on the 2-simplex. In Section 4, we discuss the stylized facts of labour allocation dynamics. In Section 5, we present our structural change model. In Section 6, we derive the structural change scenarios and show the degree of path dependency. Section 7 discusses the model assumptions. In Section 8, we show that most structural change models assume implicitly continuous and non-self-intersecting trajectories. Concluding remarks are provided in Section 9.

2. Structural dynamics and standard simplexes

Most aspects of the structure of an economy can be described by a set of shares of an aggregate construct (for a general discussion, see Stijepic, 2014a). Examples 1 and 2 illustrate this fact.

Example 1. If we are interested in the distribution of income across households $j = 1, 2, \ldots, h$, we may study the shares of households j in aggregate income y. That is, we study the system y_j/y, $j = 1, 2, \ldots, h$, where y_j is the income of household j.

Example 2. Labour allocation across sectors. Assume that E is some measure of aggregate employment (e.g. the number of hours worked in the whole economy). Furthermore, let E_i denote employment in sector i (e.g. hours worked in sector i). Previous studies of labour allocation (such as those cited in Section 1) examine the dynamics of the system $\ell_i := E_i/E$, $i = 1, 2, \ldots, n$, where n is the number of sectors and ℓ_i is the employment share of sector i.

We focus on Example 2. Two facts allow us to model labour allocation on a standard simplex. First, standard structural change literature (see Section 1) assumes that $\ell_1 + \ell_2 + \cdots + \ell_n = 1$. That is, all available labour is employed in sectors $i = 1, 2, \ldots, n$ or, equivalently, E is the aggregate of sectors $i = 1, \ldots, n$, i.e. $E := E_1 + E_2 + \cdots + E_n$. This definition is not crucial for any result, since, if $E \neq E_1 + \cdots + E_n$, we can always define an auxiliary variable E_{n+1} such that $E = E_1 + \cdots + E_n + E_{n+1}$ and study the dynamics of the system $\ell_i := E_i/E$, $i = 1, \ldots, n+1$. Second, the definitions $\ell_i := E_i/E$, $i = 1, \ldots, n$, and $E := E_1 + \cdots + E_n$ imply $0 \leq \ell_i \leq 1$ for $i = 1, \ldots, n$.

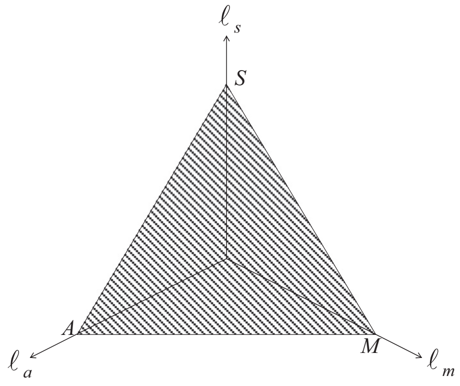

Fig. 1. The simplex Δ_2 in the Cartesian coordinate system (ℓ_a, ℓ_m, ℓ_s).

These facts reduce the set of all feasible ℓ_i significantly: the feasible ℓ_i are located on a standard simplex of dimension $(n-1)$.[4] Let $\Delta_{(n-1)}$ denote this simplex; thus, $\Delta_{(n-1)} := \{(\ell_1, \ell_2, \ldots, \ell_n) \in \mathbf{R}^n : \ell_i \geq 0,\ i = 1, \ldots, n;\ \ell_1 + \ell_2 + \cdots + \ell_n = 1\}$.

In the main part of the paper, we study a lower-dimensional case of Example 2, as defined in Assumption 1.

Assumption 1. (a) We study an economy divided into three sectors: agriculture (a), manufacturing (m), and services (s). (b) ℓ_i denotes the share of labour devoted to sector i, $i = a, m, s$. (c) The employment shares ℓ_i satisfy the following conditions:

$$\ell_a + \ell_m + \ell_s = 1 \tag{1}$$

$$0 \leq \ell_i \leq 1 \text{ for } i = a, m, s. \tag{2}$$

According to the discussion above, the employment shares of the economy defined in Assumption 1 are located on a two-dimensional standard simplex (Δ_2), where Δ_2 is defined as follows:

$$\Delta_2 := \{(\ell_a, \ell_m, \ell_s) \in \mathbf{R}^3 : \ell_i \geq 0,\ i = a, m, s;\ \ell_a + \ell_m + \ell_s = 1\} \tag{3}$$

where (ℓ_a, ℓ_m, ℓ_s) is a vector of Cartesian coordinates indicating labour allocation and $\mathbf{R}$ is the set of Real numbers.

In the remainder of this section, we recapitulate some standard geometrical concepts for the analysis of the system defined in Assumption 1.

The simplex Δ_2 is a triangle. Fig. 1 depicts Δ_2 in a three-dimensional Cartesian coordinate system with the coordinates (ℓ_a, ℓ_m, ℓ_s).

The vertices (A, M, S) and the origin (O) of the coordinate system in Fig. 1 can be expressed in Cartesian coordinates (ℓ_a, ℓ_m, ℓ_s) as follows:

$$A := (1, 0, 0) \tag{4}$$

$$M := (0, 1, 0) \tag{5}$$

$$S := (0, 0, 1) \tag{6}$$

[4] On simplexes, see, e.g., Border (1985, p. 20), and Munkres (1984, p. 2).