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Forecasting the yield curve with the arbitrage-free dynamic Nelson–Siegel model: Brazilian evidence

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Abstract

We assess the extent to which the imposition of a no-arbitrage restriction on the dynamic Nelson–Siegel model helps obtaining more accurate forecasts of the term structure. For that purpose, we provide an empirical application based on a large panel of Brazilian interest rate future contracts and test for differences in forecasting performance among alternative benchmark specifications including the random walk, vector autoregressions, and the dynamic Nelson–Siegel. We show empirically that the arbitrage-free Nelson–Siegel model is able to outperform all other benchmark models when longer forecasting horizons are taken into account.

Keywords: Yield curve; Arbitrage-free Nelson–Siegel model; Dynamic factor models; Kalman filter

Resumo

Neste artigo avaliamos em que medida a imposição de uma restrição de não arbitragem na versão dinâmica do modelo de Nelson–Siegel ajuda a obter previsões mais precisas da estrutura a termo. Para isso, realizamos uma aplicação empírica envolvendo um amplo conjunto de taxas de juros de contratos de DI-futuro negociados na BM&F Bovespa. Os resultados são comparados com os modelos competidores mais amplamente usados, incluindo o *random walk*, vetores autorregressivos e o modelo dinâmico de Nelson–Siegel. Os resultados encontrados mostram evidências de que o modelo de Nelson–Siegel com condição de não arbitragem é capaz de superar os modelos benchmarks quando se consideram horizontes de previsão mais longos em todo o espectro de maturidades analisadas.

Palavras chave: Curva de juros; Modelo Nelson–Siegel livre de arbitragem; Modelos de fatores dinâmicos; Filtro de Kalman

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1. Introduction

There has been growing interest in the ability to forecast the behavior of the term structure of interest rates. Such forecasts are of paramount importance for macroeconomists, financial economists, and fixed income managers since bond portfolio optimization, risk management, and pricing of financial assets and their derivatives rely heavily on interest rate forecasts. Moreover, these forecasts are widely used by financial institutions, regulators, and institutional investors to develop macroeconomic scenarios.

One of the most popular approaches to forecasting the yield curve is the dynamic version of the Nelson and Siegel (1987) model proposed by Diebold and Li (2006) (hereafter DNS). Existing evidence suggests that these specifications are remarkably well suited both to fit the term structure and to forecast its movements. Vicente and Tabak (2008) compared a Gaussian affine model with Diebold and Li model for Brazilian data and concluded that the latter model is slightly superior in terms of yield curve forecasts. Vereda et al. (2008) employ a VAR approach to forecast the term structure of interest rates and find that incorporating macro variables can improve forecasting performance, especially for longer-term forecasts. Almeida et al. (2009) obtained good forecasting results using an expanded version of the Nelson–Siegel model proposed by Svensson (1994) to accommodate additional nonlinearities in emerging market data. de Rezende and Ferreira (2013) proposed a five factor version of the Nelson–Siegel model and showed that it improves in-sample fit, however, out-of-sample results favored more parsimonious models. In Caldeira et al. (2010) the Nelson–Siegel model is cast in state-space form, and the parameters are simultaneously an efficiently estimated using the Kalman filter.[1]

Despite the large empirical evidence favorable to the DNS approach, one drawback is that it fails on an important theoretical dimension: it does not impose restrictions to prevent riskless arbitrage opportunities, as shown in Björk and Christensen (1999). This drawback is relevant, since many financial applications that rely on interest rate modeling such as the pricing of interest-rate-linked assets require an arbitrage-free setting. This difficulty motivated Christensen et al. (2009, 2011) to develop an arbitrage-free version of the DNS model (hereafter AFNS), thus overcoming the theoretical weakness of the original model specification.

The AFNS model of Christensen et al. (2009, 2011) has many appealing features. First, it preserves the desirable economic interpretation of the three-factor model of time-varying level, slope and curvature of the DNS specification. Second, AFNS ensures lack of arbitrage opportunities with a more parsimonious structure in comparison to general affine arbitrage-free models such as those considered in Duffie and Kan (1996) and Duffee (2002). More specifically, (Christensen et al., 2011) show that to achieve these desirable properties, one only needs to add an yield-adjustment term containing the necessary restrictions to the DNS specification.

That being said, an immediate question arises: is the no-arbitrage imposition helpful for forecasting purposes? This question is, indeed, very controversial. First, as we shall see in Section 2.2, the yield-adjustment term of the AFNS model puts no restriction on the dynamics of the yields and Joslin et al. (2011) show theoretically that no-arbitrage conditions cannot improve forecasts of the risk factors. In other words, the imposition of no-arbitrage delivers yield-adjustment terms that vary with maturity but are constant over time. This suggests that the inclusion of the yield-adjustment term is unlikely to provide forecasting gains. In particular, Duffee (2011) points out that imposition of no-arbitrage based on cross-section restrictions is irrelevant for forecasting.[2] However, empirical work has found predictive gains from imposing no-arbitrage. For instance, Ang and Piazzesi (2003), Favero et al. (2012) and Moench (2008) find that imposition of no-arbitrage often improves VAR forecasts, while Almeida and Vicente (2008) corroborate these results using polynomial models. Gimeno and Marqués (2009) and Christensen et al. (2011) used different data sets and found that imposition of no-arbitrage leads to substantial forecast improvements. Diebold and Rudebusch (2013) point out that, despite its time constancy, the yield-adjustment term can act as a bias correction and thus produce forecast improvements.[3] However, there is no clear-cut theoretical result showing that no-arbitrage restrictions improve forecasts, implying that all empirical results discussed above are data and model dependent. Thus, additional empirical

[1] Diebold and Rudebusch (2013) and the references therein present international evidence on the forecasting ability of DNS-type models.

[2] Coroneo et al. (2011) and Nyholm and Vidova-Koleva (2012) also found that the imposition of no-arbitrage adds little to forecasting accuracy.

[3] Moreover, Carriero and Giacomini (2011) show that the imposition of no-arbitrage restriction is important specially when an economic measure of accuracy is taken into account.