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Research Paper

Analysis of edge-to-center settlement ratio for circular storage tank foundation on elastic soil

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ABSTRACT

This paper presents a semi-analytical, elastic analysis model for a uniformly-loaded circular tank foundation resting on soil. The soil is assumed to behave as a linear-elastic material in a semi-infinite half space, and the foundation is modeled as a thin circular plate. The governing differential equations are derived based on variational principles and solved semi-analytically. Model validation was achieved by comparing results from the present analyses with those from finite element analysis. A parametric study highlights important insights for the effects of plate diameter, plate thickness, and stiffness ratio of plate to soil on differential settlement of the tank foundation.

1. Introduction

Large circular tanks, such as reinforced concrete water tanks or steel oil storage tanks, are widely used in many industries to store liquid. These tanks transmit the load from the liquid and self-weight to a foundation. Foundation settlements of the circular tanks are classified into three types: uniform settlement, planar tilt, and differential settlement. Excessive uniform settlement and planar tilt may cause operational problems related to serviceability but no threat to structural integrity of a tank. On the other hand, an excessive differential settlement is very damaging to the structure and often leads to ultimate limit states such as rupture of the bottom plate or rupture of the shell-bottom plate connection [1,2]. The differential settlement can be further subdivided into dish-shaped settlement and localized depressions [1]. When assessing tolerable deflections of superstructures, an angular distortion α is usually preferred over the differential settlement Δw because the same differential settlement causes different degrees of distortion for different span lengths [3]. In the case of a circular tank foundation, the span length can be taken as the radius r_p of the tank bottom plate when the differential settlement is defined as the difference in settlements between the center and the edge of the bottom plate (refer to Fig. 1). The angular distortion α is then defined as

$$\alpha = \frac{\Delta w}{r_p} \tag{1}$$

1.1. Tolerable deflection criteria

Tolerable deflection criteria exist for both steel and concrete tank foundations. The tolerable angular distortion values proposed in literature for steel and concrete tank foundations are summarized in Table 1.

In the case of steel tank foundations, the criteria were developed by examining available data from case histories of tank settlements and damages [1,2,4]. The tolerable α values for steel tank foundations available in literature range from 0.020 to 0.050 (1/50 to 1/20). However, unlike steel tank foundations, very few studies are available for the tolerable angular distortion values of concrete tank foundations. The tolerable α values for concrete tank foundations available in literature [5,6] range from 0.002 (1/500) to 0.0033 (1/300), which is about one tenth of those for steel tank foundations.

1.2. Published analytical models

Many analytical models have been proposed to predict the displacement of a beam-type foundation resting on an elastic soil under a given load. For example, Winkler [7] presented a simple model for the elastic foundation to study the behavior of a longitudinal sleeper for railway [8]. The Winkler model assumes that a uniformly distributed load q and the displacement w of the foundation have a linear relationship with a coefficient of subgrade reaction k_s (i.e., $q = k_s w$). Although the Winkler model is easy to implement due to its simplicity, it yields uniform settlement across the foundation for a given k_s and

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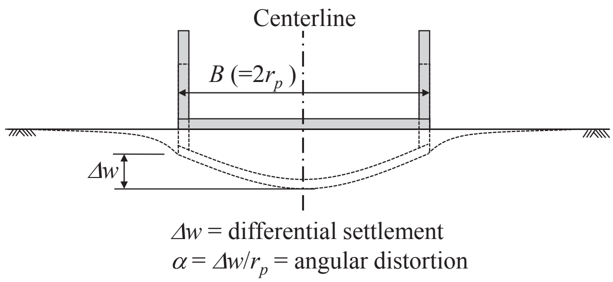

Fig. 1. Differential settlement and angular distortion of a circular plate.

Table 1
Published tolerable angular distortion values for design of storage tanks.

Tank Type	Tolerable angular distortion α_{tol} $(=\Delta w_{tol}/r_p)$	References
Steel	0.02	Langeveld (1974)
	0.022–0.040	Hayashi (1973)
		Guber (1974)
	0.044	Rinne (1963)
	0.044	Marr et al. (1983)
	0.040	Green and Hight (1974)
	0.036	Sullivan and Nowicki (1974)
	0.040–0.050	Rosenberg and Journeaux (1982)
	0.031	API (2001)
	0.050	D'Orazio and Duncan (1987)
Reinforced Concrete	0.002–0.0033	USACE (1990)
	0.0033	Brannan (2012)

therefore is not appropriate for the analysis for differential settlement of a circular tank foundation. Additionally, displacements are zero outside the loaded region from the Winkler model. In order to overcome the limitations of the Winkler model, Vlasov and Leont'ev [9] presented a two-parameter model for analysis of beams on elastic soil, underlain by a rigid layer. Subsequently, Vallabhan and Das [10] proposed an elastic analysis method for a circular plate sitting on a homogenous layer with a finite depth based on variational principles. Vallabhan and Das [10] then compared the results from their elastic analyses against those from finite element analyses (FEA) for a circular tank foundation resting on a thin soil layer, where the thickness of the soil layer was one half the diameter of the tank foundation. Although the comparison in their study shows reasonable agreement for the thin layer of soil, the discrepancy of results between the two analyses becomes larger when the thickness of the soil layer increases. Furthermore, when Poisson's ratio of soil approaches 0.5, Vallabhan and Das's [10] analysis method yields much stiffer response than FEA.

In addition to the aforementioned studies, many researchers have performed elastic analyses to study behavior of a circular plate under an applied pressure using various methods such as analytical-based method [11–16], finite element method [17–18], finite difference method [19], or other numerical methods [20]. However, most of these analyses focused on investigations of variations of flexural bending moment in a plate [12–14,16,18,20], distributions of contact pressure between the plate and the elastic medium [11,12,16–18], or estimations of center settlement [12–14,18]. Some studies did investigate the differential settlement between the center and edge of the plate but only for very limited cases [15–20].

For cases of perfectly rigid and perfectly flexible conditions, there exist exact analytical solutions to circular loaded area. Gerrard and Harrison [21] presented an analytical solution that provides stresses and displacements in the soil below a rigid circular foundation. The settlement w beneath the rigid circular foundation resting on an elastic soil under a uniformly distributed load q is given as:

$$w = \frac{\pi}{4}\frac{qB}{E_s}(1-\nu_s^2) \tag{2}$$

where B = plate diameter; E_s = Young's modulus of soil; ν_s = Poisson's ratio of soil; and q = uniformly distributed load. Similarly, Ahlvin and Ulery [22] presented analytical solutions for determining stress and displacement fields of the soil below a perfectly flexible circular foundation. The center (w_{center}) and edge (w_{edge}) settlements of the flexible circular foundation are given as:

$$w_{center} = \frac{qB}{E_s}(1-\nu_s^2) \tag{3}$$

$$w_{edge} = \frac{2}{\pi}\frac{qB}{E_s}(1-\nu_s^2) \tag{4}$$

According to Eqs. (3) and (4), the edge-to-center settlement ratio w_{edge}/w_{center} of the perfectly flexible foundation is equal to $2/\pi$ (= 0.637); whereas, that of the perfectly rigid foundation is, of course, 1, regardless of the type of tank (steel or concrete). This indicates that the edge-to-center settlement ratio of a large circular storage tank foundation resting on homogeneous soil will likely be within those ranges (0.637–1) for linear elastic behavior, depending on flexibility of the foundation.

1.3. Contribution of this study

Although estimation of differential settlement of flexible foundations is a very important tank design consideration, there is a lack of readily-available analysis tools that can be used to compute the differential settlement under various plate dimensions and soil stiffnesses. One end of the spectrum is represented by the perfectly rigid/perfectly flexible cases [e.g., Eqs. (2)–(4)]. While these methods are straightforward and quick to use, they do not take plate dimensions into considerations and only serve as upper and lower boundary solutions. FEA represents the other end of the spectrum. FEA is a powerful method which can be performed to obtain center and edge settlements of the foundation for the given soil and plate properties. However, FEA typically requires larger computational effort than analytical-based methods and may not be a plausible or cost-effective option in an early stage of a design for routine projects. Thus the need exists for an analysis method to calculate differential foundation settlement for circular tanks that is analytically rigorous, versatile, and production-oriented.

This paper presents a continuum-based, semi-analytical, elastic analysis model for a uniformly-loaded circular tank foundation resting on a soil, hereafter referred to as the "semi-analytical method." The soil is assumed to behave as a linear elastic material in a semi-infinite half space, and the tank foundation is modeled as a circular plate with a finite thickness for any material (*i.e.*, steel to concrete) which is assumed to behave as linear elastic material. The governing differential equations are derived based on energy principles and calculus of variations. The analysis effectively captures complex nature of the soil-structure interactions and produces settlement profiles of the tank foundation and the soil below it in a few seconds using a modern personal computer.

The authors first present the mathematical formulation and the derivation of the differential equations using variational principles. This paper then describes the method of analysis and compares the results from the present study with those from the literature and the FEA. Finally, parametric studies with a focus on an edge-to-center settlement ratio of the circular plate are performed and some important insights of the plate-soil interactions are highlighted.

2. Method of analysis

2.1. Problem definition

The analysis considers a circular plate of diameter B (= $2r_p$, where r_p