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Research Paper

# Nonlinear flow behavior through rough-walled rock fractures: The effect of contact area

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ABSTRACT

A computed scanning technology was applied to measure fracture tomography, and three-dimensional fracture meshes were established. Water flow experiments and simulations on fractures were conducted. Results demonstrate both Forchheimer and Izbash equations can capture the nonlinear flow behavior. As Reynolds number increases, contact obstacles and roughness of fracture increase the complexity of the velocity distribution by generating eddies or back flow. Nonlinear coefficient in Forchheimer equation increases by 1–5 orders of magnitude, ranging from $6 \times 10^{10}$ to $5 \times 10^{15}$. A critical Reynolds number is proposed to quantify the onset of nonlinear flow. Further quantitative analysis is conducted regarding flow path tortuosity.

## 1. Introduction

The investigation of fluid flow and solute transport in fractured rocks is important for engineering applications, such as oil shale mining, shale gas reservoir fracturing and production, migration of contaminant control, and hazardous waste isolation [36,26,28]. However, the flow process is still not fully understood. In particular, little is known about the nonlinear flow regime that occurs under special circumstances.

A parallel plate model is commonly used to describe Darcy flow behavior through rock fractures, which yields the classical cubic law [3,36]:

$$-\frac{\Delta P}{L} = \frac{12\mu Q}{we^3} \tag{1}$$

where $L$ [L] is the fracture length, $e$ [L] is the fracture aperture, $w$ [L] is the fracture width, $\mu$ [$ML^{-1}T^{-1}$] is the viscosity of the fluid, $Q$ [$L^3T^{-1}$] is the injection flow rate, and $\Delta P$ [$ML^{-1}T^{-2}$] is the pressure difference between the inlet and outlet of the fracture. Owing to the complexity of fracture geometries, the validity of the cubic law to single-rock fractures has been investigated. Dippenaar et al. [12] suggested that the surface roughness and contact areas of the fracture might induce turbulent flow at an earlier stage of pressure vs. flowrate curves; thus, the cubic law was found to apply to a fracture with smooth walls under low flow velocity.

However, unlike cubic law, nonlinear flow behavior through fracture could occur as a result of inertial losses [37]. Schrauf and Evans [21] and Zhang et al. [34] found that the nonlinear flow could be triggered by enhancing the inertial effect as the Reynolds number (*Re*) increased.

Chen et al. [9] observed three types of nonlinear flow induced by inertia, fracture dilation and solid-water interaction. The first type could be described by two mathematical models, the Forchheimer equation (Eq. (2)) and the Izbash equation (Eq. (3)) as follows [34]:

$$-\frac{\Delta P}{L} = AQ + BQ^2 \tag{2}$$

$$-\frac{\Delta P}{L} = \lambda Q^m \tag{3}$$

where $A$ [$ML^{-5}T^{-1}$] and $B$ [$ML^{-8}$] are the linear and nonlinear coefficients describing energy losses due to viscous and inertial effects in the Forchheimer equation, respectively. $\lambda$ and $m$ are determined by the regression analysis of flow tests in the Izbash equation. Zoorabadi et al. [38] conducted laboratory experiments on rough fractures with standard *JRC* profiles using a self-designed test apparatus. The results confirmed that both Forchheimer and Izbash equations could adequately describe the nonlinear flow behavior. Similar results were also observed by Lucas et al. [18].

Nonlinear flow occurs as a result of the inertial effect induced by complex fracture geometry. Aperture inhomogeneous distribution of fractures causes the flow in tortuosity, and results in a flow rate lower than that predicted by the cubic law [25]. Brown [3] reported that

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channeled flow paths arose in high-aperture regions. Cardenas et al. [7] studied the effect of roughness on eddy flow generation with flow simulations in a series of 2D or 3D models of rough fracture, as did Crandall et al. [10], Zou et al. [39] and Wang et al. [27]. And Wang et al. [27] emphasized that the secondary roughness was relevant to nonlinear flow behavior.

However, most of these investigations have focused on the effect of variable aperture distributions and surface roughness of fracture for water flow [3,29]. In nature, fracture surfaces have some degree of contact, and the flow regime is expected to be substantially different [5]. Tsang [25] showed that the increasing tortuosity of the flow paths was the result of the fracture contact area. However, the effect of contacts on nonlinear flow is not clear.

The flow characteristics through rough-walled fracture were investigated by experimental and numerical methods. Johnson et al. [22] designed the transparent fracture intersection to observe flow and mixing inside fracture, and numerical method further improved predictions of these phenomena. The same experimental and numerical methods were applied in Kishida et al. [14]. Develi and Babadagli [11], Babadagli et al. [2] presented the transparent fracture replicas was made to study visually single phase flow. Brown et al. [5] simulated fracture deformation under stress condition. Kumara and Indraratna [16] presented a new two-dimensional flow model for deformable fracture surfaces to predict the volumetric flow. Kang et al. [17] conducted anomalous transport simulation through a rough-walled fracture with normal stress.

In the present study, we expand on the previous work by conducting laboratory experiments of rough-walled rock fractures. A number of numerical models verified by the experimental results were further employed to study nonlinear flow characteristics under normal deformations. The effect of the contact ratio on the nonlinear flow behavior was elucidated.

## 2. Measurement of rock fractures

### 2.1. Preparation of rock fractures

To obtain the rock fractures with different morphology, a number of intact granite blocks were split using the Brazilian indirect tensile test into two halves with dimensions of 150 mm × 150 mm × 75 mm. Five representative rock fractures with different roughness were selected and marked as Fr1, Fr2, Fr3, Fr4 and Fr5.

Highly transparent replicas of original fractures were produced to visualize the flow pattern in the experimental process. Fracture replica creating method proposed by Develi and Babadali [11] was used to produce upper and lower fracture replica. The detailed process is shown in Fig. 1. Aerosol mold releaser was first sprayed on the original rock fracture surface to avoid the resultant silicone from being pasted on the surface when separating the silicone from the surface after curing. A few minutes later, silicone mold-making rubber was mixed using the silicone and curing agent and then poured onto the fracture surface. After approximately 2 h, the silicone mixture was cured and then taken out. The silicone mold was obtained, which had the same surface morphology as the original fracture. Transparent epoxy resin was casted on the silicone mold again. After 24 h, a transparent fracture replica was pealed off the silicone cast model, and then was trimmed and polished as in Fig. 1.

### 2.2. Measurement of fracture geometry

Different roughness measurement techniques for fractures have been presented in earlier works [11,9]. The advanced stereotopometric scanning system 3D CaMega (BoWeiHengXin Inc., China) (see Fig. 2) was used to map the fracture geometry. The device offers the advantages of high precision, favorable repeatability, and high measurement speed [8]. The resolution of a sampling point obtained by the scanning system is ± 25 μm, which is defined as the error of 3D space along the $x$, $y$ and $z$ directions. Roughness measurements on the 150 mm × 150 mm lower and upper halves of five fracture replicas were performed. 3D discretization of the fracture surface is shown in Fig. 3.

The fracture is fixed by matching the upper and lower surfaces. The gap still exists and provides channels for water flow, due to surfaces damage during rock block tensile test. To numerically investigate flow behaviors in fracture, it is necessary to measure the fracture geometry. The fracture void space was measured according to the approach of Tatone et al. [24]. The aperture frequency distribution of the five fractures is plotted in Fig. 4, and the mean and standard deviation of aperture are listed in Table 1.

## 3. Properties of fractures

### 3.1. Mathematical description of fracture surfaces

Several methods have been proposed in the literature, including the joint roughness coefficient (*JRC*), conventional statistical parameters [8], and fractal dimension [35,31,32]. One of the most important macroscopic roughness parameters in describing the fracture roughness is the fractal dimension ($D_v$), which is calculated in individual 2D profiles extracted from the surfaces. Kulatilake et al. [15] noted that the morphology of 2D profiles was self-affine rather than self-similar and the fractal dimension of self-affine profiles could be evaluated by the variogram method. The variogram function is defined as:

$$2\gamma(h) = \frac{1}{N}\sum_{i=1}^{N}[Z_{i+1}-Z_i]^2 \tag{4}$$

where $\gamma(h)$ [L$^2$] is the semi-variogram, $Z_i$ [L] and $Z_{i+1}$ [L] are the heights of the 2D profile from the baseline, and $N$ is the number of pairs of $Z$ at a lag distance $h$ [L] between them. $\gamma(h)$ can be simplified as a power-law function in the self-affine profile as $h \to 0$:

$$2\gamma(h) = K_v h^{2H} \tag{5}$$

where $K_v$ is a proportionality constant and $H$ is the Hurst exponent, which is related to the fractal dimension by $D_v = 2 - H$. However, Eqs. (4) and (5) cannot be used to directly calculate $D_v$. The $D_v$ should be written in logarithmic form:

$$\log(2\gamma(h)) = 2(2-D_v)\log h + \log K_v \tag{6}$$

so that the $D_v$ can be obtained by linear regression analysis.

For each fracture surface, a total of 31 sectional profiles, parallel to the flow direction at intervals of 5 mm, were extracted to calculate the fractal dimension ($D_v$) based on variogram method. The accurate fractal dimension could be determined using Eq. (6) as long as $h$ was in a certain range, according to Kulatilake et al. [15] and Brown [4] studies. It was suggested that the equation of $hd = 1.76$ ($d$ as data density) was used to estimate the minimum $h$ value. Hence, the minimum $h$ value was set to 2, for which $d = 1$ in fractures. Nine $h$ values using an increment factor of 1.2 starting from 2 were calculated for the corresponding $\gamma(h)$. 151 points were used to calculate $D_v$ in each sectional profile using Eq. (6). The $D_v$ of all sectional profiles in each fracture was averaged.

Table 1 indicates that the mean fractal dimension ($D_v$) of upper and lower surfaces varies from 1.40 to 1.54. The lowest value of $D_v$ is 1.407 for the specimen Fr1, while the highest value is 1.535 for the Fr2. Kulatilake et al. [15] pointed out the fractal dimension ($D_v$) was in range of 1.0 and 1.7. The calculated $D_v$ was consistent with the Kulatilake et al. [15] range. In addition, the root-mean-square of 2D fracture profile ($R_a$) and profile wall slope ($R_q$) [10] was introduced to evaluate the roughness. The calculation formulas can be written as: