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Research Paper

Cavity expansion in unsaturated soils of finite radial extent

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ABSTRACT

This study presents a new cavity expansion solution in unsaturated soils of finite radial extent. The solution is formulated for spherical and cylindrical cavities subjected to two boundary conditions under the drainage condition where the contribution of suction to the effective stress is constant. It is found that samples with a higher suction value are under more influence of the boundary size effect and this is more obvious with higher mean net stress and higher void ratio. Potential application of the solution in calibration chamber studies for in-situ tests in unsaturated samples is discussed.

1. Introduction

Cavity expansion theory is a very useful analytical tool in the interpretation of in-situ tests for characterizing engineering properties of soils, such as the cone penetration test (CPT) (e.g. [34,39,29,20,21]) and the pressuremeter test (PMT) (e.g. [19,40,33]).

The focus of most cavity expansion studies in the geotechnical engineering literature has been on saturated soils. It is not unusual that unsaturated soil layers are encountered in which CPTs and PMTs are performed. Unsaturated soils are more complex than their saturated or dry counterparts in that suction can significantly influence the engineering properties. This has motivated the development of the cavity expansion analysis in unsaturated soils, though studies are very limited [31,32,18,26,36]. The solutions of Schnaid et al. [31] and Schnaid et al. [32] were based on that of Carter et al. [2] where small strain was assumed in the plastic region. Muraleetharan et al. [18] extended the equations of Vesic [34] for cylindrical cavities to take into account suction. Again the elastic deformation was ignored in the plastic zone which was defined by the Mohr-Coulomb model. Russell and Khalili [26] considered large deformation in the elastic-plastic zone which was defined by a bounding surface plasticity soil model. They presented spherical and cylindrical cavity expansion solutions in two soils using the similarity technique [7,8]. Yang and Russell [36] extended the solution of Russell and Khalili [26] to incorporate the full coupling of void ratio, degree of saturation, suction and effectives stress. They studied the effects of drainage conditions and hydraulic hysteresis on the pressure required to expand a cavity in a soil where suction hardening was present. The general conclusion of these studies is that suction could have a great effect on the results, which is consistent with experimental studies of PMTs (e.g. [33]) and CPTs (e.g. [11,14,22,37]).

On the other hand, in developing correlations for interpretation and validating theoretical analysis, many CPTs and PMTs are conducted in a calibration chamber where stress and strain histories, boundary condition, density and water content of the test samples can be fully controlled. In this case, the experiment is analogous to a cavity expanding in finite radial extent. There have been only a few attempts (e.g. [38,39,30,21,4]) to solve this problem and all are limited to saturated soils. Yu [38,39] presented analytical solutions for cavity expansions using the simplistic elastic perfectly plastic Mohr-Coulomb soil model. Salgado et al. [30] presented solutions of cylindrical cavities in Mohr-Coulomb soil accounting for the rotation of principle stresses around expanding cavities and variations of friction angle and dilation angle. The solution was then used to study the effect of the boundary size on the cavity expansion results. Pournaghiazar et al. [21] considered two boundary conditions and presented generalized cylindrical and spherical cavity expansion solutions which enable more advanced constitutive models to be incorporated. Yet the solutions are limited to small ratio between the radius of elastic-plastic boundary and the finite radial extent, where the simplification involved in the solution procedure is negligible. The limitation is relaxed in the recent study of Cheng and Yang [4] who presented an exact spherical cavity expansion solution using the auxiliary independent variable of Chen and Abousleiman [3].

It should be mentioned here that, as a special case, for unsaturated

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soils in which suction hardening is absent and when there is no dependency of the soil-water characteristic curve (SWCC) on the void ratio, the solution for saturated soils is the same as for unsaturated soils under constant suction condition as the suction is simply not involved in the formulation of the solution.

Yet, it is well known that the SWCC is void-ratio dependent and it is not uncommon to observe wetting collapse mainly as a result of the presence of suction hardening. A cavity expansion solution in unsaturated soils of finite radial extent, especially for soils where suction hardening is present, is not available so far. This study aims to fill this gap and to present a more general solution for cavity expansion in unsaturated soils of finite radial extent. The problem investigated here is strongly related to calibration chamber studies of in-situ tests, such as CPT and PMT or model pile in unsaturated soils where the boundary size effect could be significant. The results measured in calibration chambers for unsaturated soil samples may be remarkably different to the values measured in the field, especially for dense samples. Such a solution would evaluate the boundary size effect for unsaturated test samples other than having to be loose to medium dense in the laboratory controlled environments.

This study presents a new cavity expansion solution which is formulated with the aid of the auxiliary independent variable of Chen and Abousleiman [3] under the constant χs condition. The drainage condition facilitates greatly the formulation and is assumed based on the findings of Yang and Russell [36] that a constant χs condition can be assumed without loss of significant accuracy, even when the actual drainage conditions are different. Also for simplicity hydraulic hysteresis is ignored whereas its effect can be referred to the study of Yang and Russell [36]. Note that this study focuses on the aspect of cavity expanding in unsaturated soils of finite radial extent, which differs from previous studies on unsaturated soils of infinite radial extent. Also a different solution technique has to be adopted. Special attention is given to the initial values at the elastic-plastic boundary, through which the effect of finite radial extent is accounted for. To highlight the effect of suction on the limiting cavity pressure, the parameters of a soil in the literature where suction hardening is present was used to generate the results. The results are then presented in a way that can be used to study the boundary size effect. Potential application of the solution is then discussed.

This study is organized as follows: Sections 2 and 3 summarize the adopted SWCC and constitutive models, respectively. The models are similar as used in Yang and Russell [36], but are tailored to suit the aim of this study. For completeness, they are briefly described in order to describe in detail the development of the new cavity expansion solution for finite radial extent in unsaturated soils in Section 4. The results are shown in Section 5 followed by conclusions.

2. Soil-water characteristic curve

The relationship between void ratio (e), degree of saturation (S_r) and suction (s) is defined through a soil-water characteristic curve (SWCC). A simplified model of Russell and Buzzi [25] is used where hydraulic hysteresis is ignored. It comprises a main drying curve and an initial transition line between the air expulsion point and the main drying curve (hereafter referred to transition line) (Fig. 1):

$$S_r = \begin{cases} 1 & \text{for } s \leqslant s_{ex} \\ (s/s_{ex})^{\beta} & \text{for } C_2^{[\alpha/(\alpha-\beta)]} s_{ex} \geqslant s \geqslant s_{ex} \\ (s/s_{ae})^{\alpha} & \text{for} s \geqslant C_2^{[\alpha/(\alpha-\beta)]} s_{ex} \end{cases} \tag{1}$$

where s_{ae} and s_{ex} are the air entry value and air expulsion value. α is a negative constant. β is the slope of the transition line between s_{ex} and the main drying curve. s_{ae} depends on void ratio e through

$$S_{ae} = C_1 e^{-\gamma} \tag{2}$$

in which C_1 is a positive constant with units of stress and γ is a constant.

A fixed ratio between s_{ae} and s_{ex} is assumed,

$$S_{ae} = C_2 s_{ex} \tag{3}$$

in which C_2 is a constant. For fractal soils $\gamma = D_s$, where D_s is the fractal dimension of the particle size distribution, and $\alpha = D_p - 3$, where D_p is the fractal dimension of the pore size distribution [24].

3. Constitutive model

The stress-strain behavior is described by a simplified version of the bounding surface plasticity model used in Yang and Russell [36]. The model is simplified, in which the bounding surface coincides with the loading surface, and becomes therefore a conventional elastic-plastic model with varying shape of ellipse in the $p' \sim q$ plane.

Conventional $p' \sim q$ notation is used, where p' and q is the mean effective stress and the deviator stress, respectively. The corresponding strain variables are the soil skeleton volumetric strain ε_p and shear (deviatoric) strain ε_q. They are related to axial and radial stresses and strains in the usual way, where:

$$p' = \frac{\sigma_1' + 2\sigma_3'}{3},\ q = \sigma_1' - \sigma_3',\ \varepsilon_p = \varepsilon_1 + 2\varepsilon_3,\ \varepsilon_q = \frac{2}{3}(\varepsilon_1 - \varepsilon_3) \tag{4}$$

and subscripts 1 and 3 denote the axial and radial components, respectively. Compressive stresses and strains are assumed positive and volumetric strain is linked to specific volume v according to:

$$\varepsilon_p = -\ln\left(\frac{v}{v_0}\right) \tag{5}$$

where $v = 1 + e$ and v_0 is the specific volume at the reference configuration. In incremental form Eq. (5) can be rewritten as:

$$\dot{\varepsilon}_p = \frac{\dot{v}}{v} \tag{6}$$

where a superimposed dot indicates an increment. Elastic and plastic strain increments sum to give total strain increments in the usual way:

$$\begin{bmatrix} \dot{\varepsilon}_p \\ \dot{\varepsilon}_q \end{bmatrix} = \begin{bmatrix} \dot{\varepsilon}_p^e \\ \dot{\varepsilon}_q^e \end{bmatrix} + \begin{bmatrix} \dot{\varepsilon}_p^p \\ \dot{\varepsilon}_q^p \end{bmatrix} \tag{7}$$

where the superscripts e and p denote the elastic and plastic components, respectively. The stresses and strains are written in vector form $\boldsymbol{\sigma}' = [p', q]^T$ and $\boldsymbol{\varepsilon} = [\varepsilon_p, \varepsilon_q]^T$, respectively.

3.1. Effective stress

The effective stress [12,1] is expressed as:

$$p' = p_{net} + \chi s \tag{8}$$

where $p_{net} = p - u_a$ is the net stress, $s = u_a - u_w$ (u_a and u_w being the pore air and pore water pressures, respectively) and χ is the effective stress parameter attaining a value of 1 for saturated soils and 0 for dry soils.

The incremental form of the effective stress is:

$$\dot{p}' = \dot{p}_{net} + \psi \dot{s} \tag{9}$$

where $\psi = d(\chi s)/ds$ is the incremental effective stress parameter.

The equation for χ proposed by Khalili and Khabbaz [13] is adopted here as:

$$\chi = \begin{cases} 1 & \text{for } s \leqslant s_{ex} \\ (s/s_{ex})^{-\zeta} & \text{for } C_2^{[\Omega/(\Omega-\zeta)]} s_{ex} \geqslant s \geqslant s_{ex} \\ (s/s_{ae})^{-\Omega} & \text{for} s \geqslant C_2^{[\Omega/(\Omega-\zeta)]} s_{ex} \end{cases} \tag{10}$$

where Ω is a material parameter with a best fit value of 0.55, ζ is the slope of the transition line between the s_{ex} and main drying curve in a ln $\chi \sim \ln s$ plane. Fig. 1 plots the effective stress parameter against suction in the ln $\chi \sim \ln s$ plane.