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System-reliability-based design and topology optimization of structures under constraints on first-passage probability

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ABSTRACT

For the purpose of reliability assessment of a structure subject to stochastic excitations, the probability of the occurrence of at least one failure event over a time interval, i.e. the first-passage probability, often needs to be evaluated. In this paper, a new method is proposed to incorporate constraints on the first-passage probability into reliability-based optimization of structural design or topology. For efficient evaluations of first-passage probability during the optimization, the failure event is described as a series system event consisting of instantaneous failure events defined at discrete time points. The probability of the series system event is then computed by use of a system reliability analysis method termed as the sequential compounding method. The adjoint sensitivity formulation is derived for calculating the parameter sensitivity of the first-passage probability to facilitate the use of efficient gradient-based optimization algorithms. The proposed method is successfully demonstrated by numerical examples of a space truss and building structures subjected to stochastic earthquake ground motions.

1. Introduction

Finding the optimal design of a structural system with regard to safety, cost or performance is one of the most essential tasks in structural engineering practice. The optimal design should achieve major design objectives representing reliable operation and safety even under stochastic excitations caused by natural hazards such as earthquakes and wind loads. Due to inherent randomness in natural disasters, however, significant uncertainties may exist in the intensity and characteristics of the excitations. Therefore, the performance of such structural systems needs to be assessed probabilistically during the optimization process.

To deal with uncertainties effectively in structural design/topology optimization, various optimization algorithms and frameworks were developed recently. For instance, the so-called *robust* design/topology optimization algorithms [1–3] aim to reduce the sensitivity of the optimal performance of a structure with respect to the randomness of interest. By contrast, *Reliability-based* design/topology optimization [4–10] aims to find optimal solutions satisfying the probabilistic constraints on the structural performance indicators. So far, these studies have been mainly focusing on accounting for uncertainties in *static* loads representing typical load patterns of the structure. Recent studies on structural optimization considering dynamic excitations employed a small number of deterministic time histories representing possible future realizations [11,12], or focused on partial descriptors of the dynamic responses such as mode frequencies [13]. These approaches have intrinsic limitations because (1) a single or small number of sample time histories may not represent all possible realizations of stochastic excitations, and (2) it is practically impossible to assess the probabilities that the structural design does not satisfy the constraints on performances, i.e. failure probabilities using this approach. Therefore, the probabilistic prediction of structural responses based on random vibration analysis is needed in the process for optimal design.

To overcome this technical challenge, the authors recently proposed a new method for topology optimization of structures under stochastic excitations [14]. In the proposed method, an efficient random vibration analysis method based on the use of the discrete representation method [15] and structural reliability theories (see [16] for a review) were integrated within a state-of-the-art topology optimization framework. The authors also developed a system reliability-based topology optimization framework under stochastic excitations [17] to cope with *system* failure events consisting of statistical dependent component events using the matrix-based system reliability method [18]. The developed method helps satisfy probabilistic constraints on a system

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failure event, which consists of multiple limit-states defined in terms of different locations, failure modes or time points as it optimizes a structural system.

In these studies by the authors, the *instantaneous* failure probabilities of the structure were evaluated at discrete time points. However, to promote applications of design/topology optimization to engineering design practice, the first-passage probability, i.e. the probability of at least one occurrence of the failure over a time interval, needs to be estimated during the optimization process. Spence et al. [19] proposed a framework for RBDO of linear systems constrained on the first-passage probability. This approach decouples the nested reliability analysis loop from the optimization loop by solving sub-optimization problem formulated from simulation results. Bobby et al. [20] presented a simulation-based framework for topology optimization of wind-excited building structures with the consideration of the first-passage probability.

The first-passage probability helps promote the use of the proposed stochastic optimization framework for the design of the lateral load-resisting system or sizing structural elements under stochastic excitations with a finite duration such as earthquake excitations. To this end, this paper introduces a stochastic design and topology optimization method that can handle probabilistic constraints on the first-passage probability, and demonstrates the method using numerical examples.

2. Random vibration analysis using discrete representation method

In the aforementioned reliability-based design optimization framework under stochastic excitations [14,17], the authors proposed to perform random vibration analysis by use of the discrete representation method [15] in order to compute the instantaneous failure probability of the stochastic response at discrete time points. In the proposed approach, for example, a zero-mean stationary Gaussian input excitation process $f(t)$ is discretized as

$$f(t) = \sum_{i=1}^{n} v_i s_i(t) = \mathbf{s}(t)^{\mathrm{T}}\mathbf{v} \tag{1}$$

where $\mathbf{s}(t)$ $(= [s_1(t),\dots, s_n(t)]^{\mathrm{T}})$ is a vector of deterministic functions that describe the spectral characteristics of the process, and $\mathbf{v} = [v_1, v_2,\dots, v_n]^{\mathrm{T}}$ is a vector of uncorrelated standard normal random variables. Among existing methods available to develop a discrete representation model in Eq. (1), a popular one for ground excitation modeling is using a filter representing the characteristic of the soil medium and a random pulse train. For example, if a filtered white noise is used, the model in Eq. (1) is constructed as

$$f(t) = \int_0^t h_f(t-\tau)W(\tau)d\tau = \mathbf{s}(t)^{\mathrm{T}}\mathbf{v}$$
$$s_i(t) = \begin{cases} \sqrt{2\pi\Phi_0/\Delta t}\cdot\int_{t_{i-1}}^{t_i} h_f(t-\tau)d\tau & t_{i-1} < t < t_i,\ i = 1,\dots,n \\ 0 & t \leqslant t_{i-1} \end{cases} \tag{2}$$

in which $W(\tau)$ denotes the white noise process whose power spectral density function is $\Phi_{WW}(\omega) = \Phi_0$, $h_f(\cdot)$ is the impulse response function of the filter, $\Delta t = t_i - t_{i-1}$, and n denotes the number of the time intervals introduced for the given time period $(0, t)$. The details of the derivation of Eq. (2) are available in Chun et al. [14].

2.1. Response of linear system under stochastic excitations

The responses of linear systems to stochastic excitation can be determined by the convolution integral consisting of their impulse response function and the discretized input process in Eq. (1). That is, a response time history $u(t)$ of the linear system subjected to the stochastic excitation $f(t)$ is derived as

$$u(t) = \int_0^t f(\tau)h_s(t-\tau)d\tau = \int_0^t \sum_{i=1}^{n} v_i s_i(\tau)h_s(t-\tau)d\tau = \sum_{i=1}^{n} v_i a_i(t) = \mathbf{a}(t)^{\mathrm{T}}\mathbf{v} \tag{3}$$

where $h_s(\cdot)$ is the impulse response function of the linear structural system, and $\mathbf{a}(t)$ denotes a vector of deterministic basis functions

$$a_i(t) = \int_0^t s_i(\tau)h_s(t-\tau)d\tau, \quad i = 1, \dots, n \tag{4}$$

Deriving the impulse response function in a finite element setting can be computationally challenging or cumbersome. To facilitate the process, the authors proposed novel numerical procedures in Chun et al. [14].

2.2. Instantaneous failure probability of linear system under stochastic excitations

In structural reliability analysis, the probability that the outcome of a random vector $\mathbf{X}$ is located inside the failure domain Ω_f, i.e. the failure probability, is computed by an integral

$$P_f = \int_{\Omega_f} f_{\mathbf{X}}(\mathbf{x})d\mathbf{x} \tag{5}$$

where $f_{\mathbf{X}}(\mathbf{x})$ is the joint probability density function (PDF) of the random vector $\mathbf{X}$. The failure domain is defined by the area where the limit-state function $g(\mathbf{x})$, e.g. capacity minus demand, takes the negative sign. In general, computing the multi-fold integral in Eq. (5) is nontrivial or computationally challenging. Structural reliability methods such as FORM and SORM (see [16] for a review) transform the space of the random variable $\mathbf{x}$ into the uncorrelated standard normal space $\mathbf{v}$. Then, the limit-state function is approximated by a linear (FORM) or quadratic function (SORM) at the design point, often alternatively termed as the most probable failure point (MPP). For example, in FORM, the failure probability is approximated as

$$P_f = \Phi[-\beta] \tag{6}$$

where β is the reliability index, i.e. the shortest distance from the origin of the standard normal space to the linearized failure surface, and $\Phi[\cdot]$ denotes the cumulative distribution function (CDF) of the standard normal distribution. Using the discrete representation method described above, limit-state functions defined for displacement or other structural responses can be described in the space of standard normal random variable $\mathbf{v}$. For example, the instantaneous failure event E_f defined for a linear structure subjected to the Gaussian input process in Eq. (1) is given by

$$E_f(t_k, u_0, \mathbf{v}) = \{g(t_k, u_0, \mathbf{v}) \leqslant 0\}, \quad \text{where } g(t_k, u_0, \mathbf{v}) = u_0 - u(t_k) = u_0 - \mathbf{a}(t_k)^{\mathrm{T}}\mathbf{v} \tag{7}$$

where u_0 is the prescribed threshold on the displacement response. In this case, the reliability index β is computed from the geometric interpretation of the limit-state surface as a closed form expression [15]

$$\beta(t_k, u_0) = \frac{u_0}{\|\mathbf{a}(t_k)\|} \tag{8}$$

It is noted that the limit-state function in Eq. (7) is linear in this case, and thus the failure probability by Eq. (6), i.e. $P_f = \Phi[-\beta(t_k,u_0)]$ does not introduce errors caused by function approximation or require nonlinear optimization to find the design point. If the structure behaves nonlinearly or the input process is non-Gaussian, one needs to use reliability methods such as FORM or SORM to compute the failure probability approximately. Using this discrete representation method, one can reduce the computational cost of the random vibration analysis, which should be repetitively performed during the optimization processes to compute the instantaneous failure probability at each updated set of design variables.