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Tracking control design of interval type-2 polynomial-fuzzy-model-based systems with time-varying delay

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ABSTRACT

In this paper, the tracking control design for the interval type-2 (IT2) polynomial-fuzzy-model-based (PFMB) control system subject to time-varying delay situation is investigated. The tracking control system is formed of the IT2 polynomial fuzzy model representing a nonlinear system with time-varying delay, the stable reference model and the IT2 polynomial fuzzy controller. The control objective is to design a proper IT2 polynomial fuzzy controller which is capable of driving the states of the polynomial fuzzy model to track those in the reference model and the tracking performance is evaluated and improved by the H_∞ performance index. Also, to handle the uncertainty in the membership functions, the property of IT2 fuzzy sets is utilized to enhance the fuzzy controller's robustness against uncertainty. In addition, considering the effect of time-varying delay, the Lyapunov–Krasovskii functional based approach is adopted to facilitate the delay-dependent stability analysis. Stability conditions depending on the time-varying delay characteristic with the consideration of H_∞ performance are obtained in terms of sum-of-squares (SOS). Furthermore, the information of the IT2 membership functions is employed in the stability analysis to relax the stability conditions, both membership-function-independent (MFI) and membership-function-dependent (MFD) approaches are presented to develop the stability conditions. Simulation examples are presented to verify the effectiveness of the proposed tracking control approach.

1. Introduction

Analysis of nonlinear control systems is generally challenging due to their complexity in nature. An effective way to represent the dynamics of the complex nonlinear control system is the Takagi–Sugeno (T–S) fuzzy-model-based (FMB) approach (Takagi and Sugeno, 1985; Sugeno and Kang, 1988), in which a family of local linear sub-systems are adopted and then fuzzily blended together smoothly through the membership functions to describe the global behavior of the nonlinear system. Thanks to its favorable model structure in support of control design and its rigorous mathematical foundation, the stability analysis and control synthesis of T–S FMB control systems can be conducted in a systematic way.

For the T–S FMB control systems, a popular approach to investigate the stability is based on Lyapunov stability theory. From the Lyapunov stability theory, if there is a common solution exists for all Lyapunov inequalities in terms of linear matrix inequalities (LMIs), then the T–S FMB control system is guaranteed to be asymptotically stable (Wang et al., 1996; Tanaka and Wang, 2004). Given that the stability conditions of the T–S FMB control systems are in the form of LMIs, it can be solved numerically by convex programming techniques. There are fruitful research outcomes on the T–S FMB control problems, just to name a few, the works in Wang et al. (1996), Tanaka et al. (1998), Kim and Lee (2000), Tanaka and Wang (2004), Liu and Zhang (2003b, a), Teixeira et al. (2003), Fang et al. (2006) and Sala and Ariño (2007) are dedicated to relaxing the stability conditions of T–S FMB control systems; the works in Nguang and Shi (2003), Xu and Lam (2005), Lin et al. (2005) and Zhou et al. (2005) are of the H_∞ control design. Besides the stabilization problems of FMB control systems, the tracking control issues are frequently confronted in many control applications and the tracking control problems are generally considered to be more challenging than the stabilization problems. In the tracking control design, the controller is required to drive the states of the plant to track those of a stable reference model rather than just stabilize the plant (Tseng et al., 2001). Inspired by the success of applications of the FMB control approach, fuzzy tracking control technique was introduced in the work in Tseng et al. (2001) and H_∞ performance

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index was considered to evaluate the tracking performance. There are many research results achieved on various tracking control problems such as output-feedback tracking control problems (Tseng, 2006; Lian and Liou, 2006; Lin et al., 2006; Mansouri et al., 2009; Lam and Li, 2013) and sampled-data output feedback is considered in the tracking control strategy (Lam and Seneviratne, 2009).

Recently, the T–S FMB control systems have been extended to the polynomial-fuzzy-model-based (PFMB) control systems by allowing polynomial terms to exist in the model and the stability conditions of PFMB control system can be obtained in the forms of sum-of-squares (SOS) (Tanaka et al., 2009). The SOS-based stability conditions can be solved efficiently by using existing optimization techniques, for example, the SOSTOOLS (Prajna et al., 2004). The polynomial fuzzy model has more potential to represent the nonlinear dynamics than the T–S fuzzy model since when the order of the polynomial terms is 0, the polynomial fuzzy model will be reduced to a T–S fuzzy model. In the literature, the most popular type of membership functions used in the FMB/PFMB control systems is of type-1 fuzzy sets. It is well-known that the control strategies adopting type-1 fuzzy sets have been applied successfully to tackle the nonlinearities in control systems. However, the control strategies based on the type-1 fuzzy sets lack the ability to deal with the uncertainty directly. Uncertainties cannot be avoided under many situations (Mendel et al., 2006; Mendel, 2007), for example, the parameter uncertainties and different understanding of fuzzy rules from different people. To cope with the unavoidable uncertainties, the concept of footprint of uncertainty (FOU) is introduced along with the type-2 fuzzy sets (Mendel, 2007) to include the uncertainties into the type-2 membership functions. However, the complexity of the control system is increased. To alleviate the complexity of the type-2 fuzzy sets based systems, the interval type-2 (IT2) fuzzy sets (Liang and Mendel, 2000; Mendel et al., 2006) are introduced as a compromise made between the type-1 and type-2 fuzzy sets. In IT2 fuzzy sets, the secondary grade of the membership is considered as a constant instead of a secondary function in the type-2 fuzzy sets. Relevant research of system control and stability analysis have been conducted recently based on the framework of IT2 FMB/PFMB control systems can be found in Lam and Seneviratne (2008), Juang and Hsu (2009), Biglarbegian et al. (2010), Jafarzadeh et al. (2011a, b), Lam et al. (2014), Li et al. (2016), Li et al. (2015), Xiao et al. (2017a, b) and Song et al. (2017).

Time-delay appears commonly in various practical systems such as chemical processes, networked systems and communication systems (Zhao et al., 2009; Wu et al., 2011; Su et al., 2013; Yang et al., 2014; Zhu et al., 2017; Qi et al., 2018), which is generally considered as the source of poor system performance and instability. Given that there are many complex nonlinear systems subject to time delay in practical situations and FMB control approaches are effective to represent the dynamics of nonlinear systems, it is natural to investigate nonlinear systems with time delay through the corresponding FMB control approaches (Cao and Frank, 2000). Therefore, the research of the FMB control system with time-delay is of great importance and researchers dedicated considerable effects to the problems of analysis and synthesis for time-delay FMB control systems. There are two main approaches to handle the time-delay problems in the literature, namely the delay-independent (Cao and Frank, 2000, 2001; Wang et al., 2004) and the delay-dependent (Guan and Chen, 2004; Chen et al., 2005; Zhou and Li, 2005; Chen et al., 2006; Wu, 2006; Wu and Li, 2007; Lam and Leung, 2007; Gao et al., 2009; Zhang and Xu, 2009; Zhao et al., 2009; Wu et al., 2011; Su et al., 2013; Yang et al., 2014; Wu et al., 2014) approaches. For the delay-independent approach, the stability conditions include no information of the delay, which means the stability conditions are guaranteed for arbitrary time delay. On the contrary, the delay-dependent approach contains the information of the delay, which is able to achieve less conservative results than the delay-independent approach, especially when the delay time is small. Within the delay-dependent approach, there are works on the constant time delay problems (Guan and Chen, 2004; Chen et al., 2005; Zhou and Li, 2005; Wu, 2006) and time-varying delay problems (Wu and Li, 2007; Gao et al., 2009; Zhang and Xu, 2009; Wu et al., 2011; Su et al., 2013; Yang et al., 2014; Wu et al., 2014). The advantage of the time-varying approach is that the constant time-delay case can be regarded as a special case of time-varying delay FMB control systems. It is also noticed that in the works in Zhou et al. (2015) and Li et al. (2017), the time delay issues based on IT2 fuzzy sets were investigated.

Having mentioned and reviewed the previous related works, to the authors' best knowledge, there is little literature on the tracking control design of IT2 PFMB control systems with time-varying delay, which is the main motivation for this paper. In this paper, the tracking control problem for the IT2 PFMB system is investigated under time-varying delay. To begin with the investigation, the IT2 polynomial fuzzy model is first built to represent the dynamics of the time-varying nonlinear plant. In the meantime, the uncertainty is included in the IT2 membership functions of the polynomial fuzzy model. An IT2 polynomial fuzzy controller is designed to drive the states of the IT2 polynomial fuzzy model to follow those of a reference model and the tracking performance is optimized according to the H_∞ performance. The optimization of the tracking performance in the analysis is formulated as the generalized eigenvalue minimization problem (GEVP). It should be noted that due to the uncertainty contained in the IT2 membership functions of the IT2 polynomial fuzzy model, the parallel distributed compensation (PDC)-based stability analysis in Wang et al. (1996), Tanaka et al. (1998), Kim and Lee (2000), Tanaka and Wang (2004), Liu and Zhang (2003b, a), Teixeira et al. (2003), Fang et al. (2006) and Sala and Ariño (2007) cannot be applied anymore since it requires that both the fuzzy model and the fuzzy controller share exactly the same premise membership functions. When the PDC approach is not applied, the analysis result can be very conservative. To further relax the stability conditions, we suggest the membership-function-dependent (MFD) approach, in which the information of the membership functions can be included in the stability conditions in terms of SOS. On the contrary, the membership-function-independent (MFI) stability conditions do not take any information of membership functions into account, which means in the MFI approach, the stability conditions are guaranteed unnecessarily for all possible membership functions, which is the source of conservativeness. Therefore, the MFD stability conditions are more relaxed even though the computational demand is generally higher than the MFI ones (Lam, 2017). This conclusion will be verified by simulation examples in the paper.

To differentiate the proposed work with our recent published works in Song et al. (2017) and Lam (2017), we would like to emphasis the contribution of the current work as following:

(1) The tracking control design is presented in the current work while the state stabilization case is investigated in Song et al. (2017).
(2) The time-varying delay effects are also considered and addressed based on the Lyapunov–Krasovskii functional approach in the current work. In Song et al. (2017), there is no discussion on the time-varying delay effects in the control systems.
(3) The work in Lam (2017) serves as a review of recently published works in the fuzzy model based (FMB) control systems. Although the related IT2 and PFMB control systems have been discussed in Lam (2017). The tracking control issue of the IT2 PFMB control systems has not been discussed in detail as the current work does.
(4) The tracking control of the IT2 PFMB control systems subject to time-varying delay effect has not been discussed in Lam (2017).

This paper is organized as follows: In Section 2, the IT2 polynomial fuzzy model considering a time-varying delay, the IT2 polynomial fuzzy controller and the reference model are introduced. In Section 3, the stability of the IT2 PFMB tracking control system with time-varying delay is investigated and both the MFI and MFD SOS-based stability conditions are obtained. In Section 4, the simulation examples are given to show the effectiveness of the proposed approach. The conclusion is drawn in Section 5.