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# Contribution to power transformers leakage reactance calculation using analytical approach

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ABSTRACT

In this paper, classical approaches to evaluate leakage reactance in power transformers, that is, the simple analytical model, the Rogowski and the Stephens methods, are compared with each other and its limitations are exposed. To generalize and to improve the accuracy of the transformer leakage reactance evaluation, the authors propose a new analytical approach whose parameters are determined using curve fitting method from solutions obtained by 2D Finite Element Method (FEM) simulations. Since these parameters depend on the transformer windows and windings dimensions, its values are determined using per unit systems and also for typical transformer height to width ratio from 2 to 4. Consequently, the proposed method can be applied directly for the leakage reactance calculation without performing FEM simulations and for a large family of transformers with different windings configurations. The proposed method is thus compared with the classical approaches and its results show a good agreement compared with those obtained through 2D FEM simulations for any window and winding configurations.

## 1. Introduction

Leakage reactance value has a significant impact on the power transformers overall design and its calculation has been the subject of many research throughout the years [1–9]. According to [10], if this reactance value is too low, short circuit currents and forces are quite high, demanding the use of lower current density. On the other hand, if this value is too high, eddy loss in windings and stray loss in structural parts will increase, as well as load loss and winding/oil temperature rise. In both cases, the increment of copper content or the use of extra cooling arrangement will be necessary, increasing substantially the device total cost. However, the reactance estimation may represent significant cost reductions in the transformer design.

Besides costs, the reactance value must fulfill the international standards ANSI/IEEEC57.12.00 and IEC 60076-1 that mandate a tolerance of ±7.5% for two winding transformers with percentage reactance more than 2.5%, and ±10% tolerance for reactance values less than 2.5%, respectively [1]. In general, a transformer reactance can be less than 2% for small distribution transformers, and greater than 20% for large power transformers, according to its operation [10]. Due to these reasons, an accurate technique to compute the reactance value is of great interest to researchers and manufacturers.

The calculation of the transformer leakage reactance can be performed by classical analytical approaches [1–11], advanced analytical techniques [12,13], method of images [2] or by Finite Element Method (FEM) [14,15].

The simple analytical approach considers that (i) the leakage field is predominantly axial, (ii) the ampère-turn along windings are uniformly distributed and (iii) the core permeability is infinitely high [4,2]. Despite these constraints, this approach remains the most used model for computing leakage reactance due to its simplicity and ease of implementation. In fact, the leakage field is predominantly axial throughout the windings, except at the end windings, where leakage flux finds the shorter path to return and a fringing flux occurs [10]. Thus, the leakage reactance calculation must take into account both axial and radial magnetic fields components [4].

Among the solutions proposed to include the fringing flux effects on the leakage reactance calculation are the approaches of Rogowski [12] and Roth [13], which are based on single and double Fourier series to describe the magnetic vector potential throughout space [2]. Despite their good results, these methods are complicated, especially when winding heights are different. A simplified method of the Rogowski approach can be used to take into account the fringing flux effect on the leakage reactance calculation. It consists in applying to the simple analytical approach a coefficient called Rogowski factor in order to consider the radial components of the magnetic flux [10]. However,

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this approach presents good results for leakage reactance calculation only for cases in which the windings heights are equal. This limitation can be solved using the Stephens Method [6,7] which gives better results as will be shown further.

Another solution is to use the method of images, where an image of the conductors replaces boundary conditions of the problem. This method has been recently associated with analytical formulations for the magnetic vector potential and energy on the leakage reactance calculation for different windings height [2]. In this case, the associated error decreases as the number of layers of images increases, and it can provide significant gains in terms of computing time when compared to 2D FEM. Nonetheless, depending on windings geometry complexity, the number of layers of this method must be high for an accurate solution and consequently, the computing time of this approach will greatly increase.

The FEM is the most commonly used numerical method for accurate leakage reactance calculation of non-standard winding configurations and asymmetrical non-uniform ampere-turn distributions [10,16]. This method solves Maxwell's equations directly and provides an accurate solution to the problem, even for complex geometries [17]. Despite its advantages, the computational cost of this method remains high and is not recommended in the transformer design stage. For this reason, manufacturers still prefer to use simplified analytical methods, providing faster results, even with simplified geometries and, consequently, with fewer accuracy results.

Finally, there is a currently need to develop analytical models that fill the manufacturers needs in terms of computational cost and better solution accuracy, especially considering different winding configurations [18]. Thus, in this paper, a contribution to the analytical approach to compute leakage reactance in two-winding transformers is proposed. The simple analytical approach, considering only axial components of the leakage flux, is used for the calculation of the minimum leakage impedance value (vertex) and the behavior of the radial components is added to the classical model as an elliptic paraboloid function (curve fitting) of the coil heights. The curvature level of this function is specified parametrically in terms of coils and window dimensions and these parameters are obtained using a linear multiple regression function from similar 2D FEM model. The results of the proposed approach are validated using the 2D FEM results and compared with classical approaches, like the simple analytical approach, the simplified Rogowski model and the Stephens method for different winding configurations.

## 2. Classical approaches for leakage reactance calculation

In this section, classical approaches for the leakage reactance calculation in power transformers are presented. The leakage reactances calculated by the simple analytical approach, the simplified Rogowski model, the Stephens method and 2D FEM are exposed and compared with each other. For illustration purpose, these methods are applied to a transformer with ratio height to width equal to 3 and which parameters are presented in Table 1. These parameters are kept constant while varying windings height.

**Table 1**
Transformer parameters.

| Parameter | Value |
|---|---|
| Window height ($h_w$) | 0.394 m |
| Window width ($w_w$) | 0.131 m |
| Gap ($w_g$) | 0.020 m |
| Winding 1 width ($w_1$) | 0.030 m |
| Winding 2 width ($w_2$) | 0.030 m |
| Mean radius - Winding 1 ($R_{m1}$) | 0.172 m |
| Mean radius - Winding 2 ($R_{m2}$) | 0.222 m |
| Mean radius - Gap ($R_{mg}$) | 0.197 m |
| Number of turns ($N$) | 537 |
| Frequency ($f$) | 60 Hz |

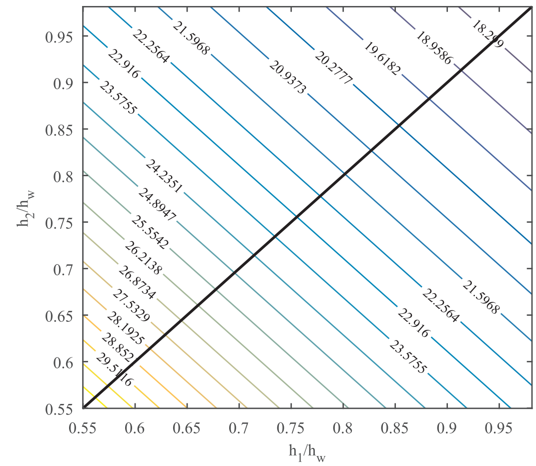


**Fig. 1.** Leakage reactance versus relative windings heights - simple analytical approach.

### 2.1. Analytical approach

The simple analytical approach assumes that the leakage magnetic field is completely axial between the two short-circuited windings and its calculation is performed only in the region between the two windings [1–5]. In this approach, the ampère-turns are uniformly distributed along the windings and the windings are treated as if they were infinitely long solenoids insofar as the magnetic field is concerned [4]. Thus, the leakage reactance is given by

$$X_{l12} = 2\pi f\frac{\mu_0 N^2}{h}\left[\frac{l_{m1}w_1}{3} + \frac{l_{m2}w_2}{3} + l_{mg}w_g\right] \tag{1}$$

where $f$ is the frequency, $\mu_0$ is the free space permeability, $N$ is the number of turns of the winding where the reactance is referred to and $h$ is the windings height. For a different windings heights configuration, $h$ is approximated by the simple average of the windings heights $h = (h_1 + h_2)/2$. The mean lengths of the two windings are represented by $l_{m1}$ and $l_{m2}$ respectively, $w_1$ and $w_2$ are the width of the windings, $w_g$ is the gap between the windings. The mean lengths are expressed as $l_{mi} = 2\pi R_{mi}$, where $i$ can be respectively 1, 2 and $g$.

Fig. 1 shows the leakage reactance calculated by the simple analytical approach for different winding heights related to the total height of the transformer window and according to the parameters presented in Table 1. In this figure, the $x$-axis represents the height of the winding 1 related to the window height ($h_1/h_w$) and the $y$-axis represent the height of the winding 2 related to the window height ($h_2/h_w$). For the case where winding heights are 80% of the total window height ($h_1/h_w = h_2/h_w = 0.8$), e.g., the total leakage reactance calculated by the simple analytical approach results in 21.6 Ω, as shown in Fig. 1.

### 2.2. Simplified Rogowski model

The simplified Rogowski model aims to take into account the effect of the fringing flux at the end windings by means of the Rogowski factor $K_r$ (< 1.0) [12] calculated as

$$K_r = 1 - \frac{c}{\pi \cdot h}\left(1 - e^{\frac{-\pi h}{c}}\right) \tag{2}$$

where $c$ is given by

$$c = w_1 + w_g + w_2 \tag{3}$$

As for the simple analytical approach, when winding 1 and winding 2 present unequal height, $h$ in (2) is replaced by the windings heights average. Then, the corrected height $h_r$ is obtained by

$$h_r = h/K_r \tag{4}$$