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State-in-mode analysis of the power flow Jacobian for static voltage stability

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ABSTRACT

In static voltage stability analysis, participation factors have been used as an index that measures the contribution of the critical mode of power flow Jacobian in the system states. However, this index as usually defined is in a mode-in-state manner that cannot adequately reflect the impact of system states on voltage collapse. In this paper we take a new state-in-mode viewpoint to study the power flow Jacobian. We express the critical mode into a weighted sum of system states, which gives rise to the definitions of state-in-mode participation factor (SIMPF) and state-in-mode sensitivity (SIMS). The SIMPF measures the contribution of a system state to the critical mode, and the SIMS measures the control sensitivity of the system state to the critical mode. The proposed SIMPF and SIMS apply to both node states and network states including active/reactive power injections and active power flows across lines. They provide new insights into the mechanism of saddle-node bifurcation and limit-induced bifurcation, two most common types of voltage instability, by revealing the role of system states. The SIMPF and SIMS can also guide the system dispatch for voltage stability enhancement. The obtained results are validated by the simulations on IEEE 118-bus system and Polish 3120-bus system.

1. Introduction

Power flow equations are the fundamental equations in power systems, which describe the balance between load and generation via power transfer over the underlying power network. Power flow Jacobian, i.e., the Jacobian matrix of the power flow equations, plays an important role in power flow analysis. It is not only an essential quantity in the Newton-Raphson iteration for finding power flow solutions, but also provides rich information for power system planning, operation and control.

In particular, the static voltage stability problem, which refers to the existence of power flow solution, is closely linked to the properties of power flow Jacobian. The singularity point of the power flow Jacobian is commonly regarded as a static voltage stability limit [1]. Under certain circumstances, the singularity point is also equivalent to the saddle node bifurcation (SNB) of the system dynamical equations [2,3]. In addition, applying modal analysis to power flow Jacobian gives rise to the concept of participation factor [4], which measures the relative contribution of a mode (defined as an eigenvalue of power flow Jacobian) to the V-Q sensitivities of load buses. The participation factor with respect to the critical mode (zero eigenvalue) has been widely used to identify the effective placement for reactive power compensators [5–7]. The participation factor considering the second-order expansion of power flow Jacobian is proposed in [8]. Also, other variants of participation factors have been developed to measure the contribution of a mode in the sensitivities with respect to generators [9,10].

In essence, the participation factor represents the influence of the critical mode in node sensitivities. It can be classified as the mode-in-state information, and we henceforth call it the "mode-in-state participation factor" (MISPF) to disambiguate. Besides the MISPF, there is another type of index called the state-in-mode participation factor (SIMPF), which measures the contribution of system states to the critical mode. The concept of SIMPF is more important than MISPF in revealing the mechanism of voltage instability, i.e., how the critical mode is formed as the system states evolve. The definition of SIMPF was first proposed in [11], which shows that the MISPF and SIMPF take the same expression by assuming the state variation direction is parallel with the right eigenvector of the critical mode. It is subsequently shown in [12] that MISPF and SIMPF are generally non-identical when considering arbitrary state variation directions. However, in [12], the concept of SIMPF has been redefined based on the dynamics context, i.e., the modes in differential equations. This raises the question of how a similar dichotomy may apply to algebraic equations such as power flow equations. In addition, some recent works [13,14] revealed that voltage collapse is closely linked to not only node states but also network states (e.g., power flow across lines). These results indicate that

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the state-in-mode idea deserves further attention for understanding the role of system states in voltage stability.

In this paper, we develop a novel state-in-mode analysis framework tailored for the power flow Jacobian. The definitions of SIMPF and state-in-mode sensitivity (SIMS) are proposed. The SIMPF and SIMS measure the contribution and control sensitivity of system states to the critical mode, respectively. The merits of the proposed indices are threefold. First, the SIMPF and SIMS apply to both node states and network states, which include active power injections, reactive power injections and active power flows across lines. Second, unlike the MISPF that are mainly designed for SNB, the SIMPF and SIMS apply to both SNB and another common type of voltage instability namely the limit-induced bifurcation (LIB), which provide a unified viewpoint for the instability mechanism. Third, the SIMPF and SIMS indicate effective directions for active power dispatch and reactive power compensation to enhance voltage stability.

The remainder of this paper is organized as follows. A brief review of the power flow model and traditional MISPF is given in Section 2. In Section 3, the state-in-mode analysis of power flow Jacobian is carried out and the definitions of SIMPF and SIMS are proposed. The results are verified by numerical simulation on IEEE 118-bus system in Section 4, and a conclusion is made in Section 5.

2. Problem formulation

2.1. Power flow equations and static voltage stability

Consider a power system with $n+1$ buses. Without loss of generality, we number the PQ buses as $\mathcal{V}_{pq}=\{1,2,\dots,d\}$, PV buses as $\mathcal{V}_{pv}=\{d+1,\dots,n\}$ and the slack bus as $\mathcal{V}_s=\{n+1\}$. For each bus i, we denote P_i, Q_i as its active and reactive power injection, and θ_i, V_i as its phase angle and voltage magnitude. In addition, denote the admittance matrix as $\boldsymbol{Y}=\boldsymbol{G}+\mathrm{j}\boldsymbol{B}\in\mathbb{C}^{(n+1)\times(n+1)}$, where the matrices $\boldsymbol{G}$, $\boldsymbol{B}$ denote the real part and imaginary part. Then, the power flow equations can be expressed as follows

$$
\begin{aligned}
P_i &= V_i^2G_{ii}+\sum_{j\in\mathcal{N}_i}V_iV_j|Y_{ij}|\sin(\theta_{ij}-\varphi_{ij}),\ i\in\mathcal{V}_{pq}\cup\mathcal{V}_{pv}\\
Q_i &= -V_i^2B_{ii}-\sum_{j\in\mathcal{N}_i}V_iV_j|Y_{ij}|\cos(\theta_{ij}-\varphi_{ij}),\ i\in\mathcal{V}_{pq}
\end{aligned}
\tag{1}
$$

where θ_{ij} represents $\theta_{ij}=\theta_i-\theta_j$; $\mathcal{N}_i$ denotes the set of adjacent buses of bus i, and $j\in\mathcal{N}_i$ means bus i and bus j are directly connected by a line; Y_{ij}, G_{ij} and B_{ij} are the (i,j) entry of $\boldsymbol{Y}$, $\boldsymbol{G}$ and $\boldsymbol{B}$, respectively; $\varphi_{ij}=-\tan^{-1}(\frac{G_{ij}}{B_{ij}})$ is the phase shift caused by line loss. The power injections P_i, Q_i in Eq. (1) are further parameterized to describe a load increase scenario

$$
\begin{aligned}
P_i &= P_{Gi}^0-P_{Li}^0+\mu(P_{Gi}^{inc}-P_{Li}^{inc}),\ i\in\mathcal{V}_{pq}\cup\mathcal{V}_{pv}\\
Q_i &= Q_{Gi}^{\lim}-Q_{Li}^0-\mu Q_{Li}^{inc},\ i\in\mathcal{V}_{pq}
\end{aligned}
\tag{2}
$$

where P_{Li}^0, Q_{Li}^0, P_{Gi}^0 denote the active power load, reactive power load and active power generation at bus i at a normal operating point, respectively; P_{Li}^{inc}, Q_{Li}^{inc}, P_{Gi}^{inc} denotes the predefined increase directions of the active power load, reactive power load and active power generation, respectively; $Q_{Gi}^{\lim}$ denotes the reactive power limit (either upper or lower limit) of the generator at bus i; and the parameter μ is the load increase coefficient.

Given the vector of active power injections $\boldsymbol{P}=[P_i]\in\mathbb{R}^n$, $\forall\, i\in\mathcal{V}_{pq}\cup\mathcal{V}_{pv}$, vector of reactive power injections $\boldsymbol{Q}_d=[Q_i]\in\mathbb{R}^d$, $\forall\, i\in\mathcal{V}_{pq}$, vector of PV bus voltages $\boldsymbol{V}_g=[V_i]\in\mathbb{R}^{n-d}$, $\forall\, i\in\mathcal{V}_{pv}$ and slack bus voltage $V_{n+1}=V_{n+1}^0$, $\theta_{n+1}=0$. Denote $(\boldsymbol{\theta},\boldsymbol{V}_d)$ as the corresponding power flow solution, where $\boldsymbol{\theta}=[\theta_i]\in\mathbb{R}^n$, $\forall\, i\in\mathcal{V}_{pq}\cup\mathcal{V}_{pv}$ denotes the vector of non-slack bus angles and $\boldsymbol{V}_d=[V_i]\in\mathbb{R}^d$, $\forall\, i\in\mathcal{V}_{pq}$ denotes the vector of PQ bus voltages. Linearizing Eq. (1) at $(\boldsymbol{\theta},\boldsymbol{V}_d)$ gives

$$
\begin{bmatrix}\Delta\boldsymbol{P}\\ \Delta\boldsymbol{Q}_d\end{bmatrix}=\begin{bmatrix}\boldsymbol{J}_{P\theta} & \boldsymbol{J}_{PV}\\ \boldsymbol{J}_{Q\theta} & \boldsymbol{J}_{QV}\end{bmatrix}\begin{bmatrix}\Delta\boldsymbol{\theta}\\ \Delta\boldsymbol{V}_d/\boldsymbol{V}_d\end{bmatrix}=\boldsymbol{J}_{pf}\begin{bmatrix}\Delta\boldsymbol{\theta}\\ \Delta\boldsymbol{V}_d/\boldsymbol{V}_d\end{bmatrix}
\tag{3}
$$

where the notation "/" means entry-wise division for two vectors. The matrix $\boldsymbol{J}_{pf}\in\mathbb{R}^{(n+d)\times(n+d)}$ is the power flow Jacobian. The entries of $\boldsymbol{J}_{P\theta}\in\mathbb{R}^{n\times n}$, denoted $(\boldsymbol{J}_{P\theta})_{ij}=\frac{\partial P_i}{\partial\theta_j}$, $i,j\in\mathcal{V}_{pq}\cup\mathcal{V}_{pv}$, take values as

$$
(\boldsymbol{J}_{P\theta})_{ij}=\begin{cases}\sum_{k\in\mathcal{N}_i}V_iV_k|Y_{ik}|\cos(\theta_{ik}-\varphi_{ik}),\ i=j\\ -V_iV_j|Y_{ij}|\cos(\theta_{ij}-\varphi_{ij}),\ i\neq j.\end{cases}
\tag{4}
$$

The entries of $\boldsymbol{J}_{PV}\in\mathbb{R}^{n\times d}$, denoted $(\boldsymbol{J}_{PV})_{ij}=V_j\frac{\partial P_i}{\partial V_j}$, $i\in\mathcal{V}_{pq}\cup\mathcal{V}_{pv}$, $j\in\mathcal{V}_{pq}$, take values as

$$
(\boldsymbol{J}_{PV})_{ij}=\begin{cases}\sum_{k\in\mathcal{N}_i}V_iV_k|Y_{ik}|\sin(\theta_{ik}-\varphi_{ik})+2V_i^2G_{ii},\ i=j\\ V_iV_j|Y_{ij}|\sin(\theta_{ij}-\varphi_{ij}),\ i\neq j.\end{cases}
\tag{5}
$$

The entries of $\boldsymbol{J}_{Q\theta}\in\mathbb{R}^{d\times n}$, denoted $(\boldsymbol{J}_{Q\theta})_{ij}=\frac{\partial Q_i}{\partial\theta_j}$, $i\in\mathcal{V}_{pq}$, $j\in\mathcal{V}_{pq}\cup\mathcal{V}_{pv}$, take values as

$$
(\boldsymbol{J}_{Q\theta})_{ij}=\begin{cases}\sum_{k\in\mathcal{N}_i}V_iV_k|Y_{ik}|\sin(\theta_{ik}-\varphi_{ik}),\ i=j\\ -V_iV_j|Y_{ij}|\sin(\theta_{ij}-\varphi_{ij}),\ i\neq j.\end{cases}
\tag{6}
$$

The entries of $\boldsymbol{J}_{QV}\in\mathbb{R}^{d\times d}$, denoted $(\boldsymbol{J}_{QV})_{ij}=V_j\frac{\partial Q_i}{\partial V_j}$, $i,j\in\mathcal{V}_{pq}$, take values as

$$
(\boldsymbol{J}_{QV})_{ij}=\begin{cases}-\sum_{k\in\mathcal{N}_i}V_iV_k|Y_{ik}|\cos(\theta_{ik}-\varphi_{ik})-2V_i^2B_{ii},\ i=j\\ -V_iV_j|Y_{ij}|\cos(\theta_{ij}-\varphi_{ij}),\ i\neq j.\end{cases}
\tag{7}
$$

Note that the power flow Jacobian $\boldsymbol{J}_{pf}$ in Eq. (3) is slightly different from the conventional power flow Jacobian, say $\boldsymbol{J}_{pf}^{conv}$, which is defined by

$$
\begin{bmatrix}\Delta\boldsymbol{P}\\ \Delta\boldsymbol{Q}_d\end{bmatrix}=\boldsymbol{J}_{pf}^{conv}\begin{bmatrix}\Delta\boldsymbol{\theta}\\ \Delta\boldsymbol{V}_d\end{bmatrix}.
\tag{8}
$$

It follows that $\boldsymbol{J}_{pf}=\boldsymbol{J}_{pf}^{conv}\mathrm{diag}\{\boldsymbol{I}_n,\boldsymbol{D}_V\}$, where $\boldsymbol{D}_V=\mathrm{diag}\{V_i\}\in\mathbb{R}^{d\times d}$, $\forall\, i\in\mathcal{V}_{pq}$ and $\boldsymbol{I}_n\in\mathbb{R}^{n\times n}$ denotes the identity matrix. It implies that the determinants of $\boldsymbol{J}_{pf}$ and $\boldsymbol{J}_{pf}^{conv}$ have the same sign, and hence $\boldsymbol{J}_{pf}$ and $\boldsymbol{J}_{pf}^{conv}$ provide the same indication of voltage instability, which will be detailed later. Here we adopt $\boldsymbol{J}_{pf}$ as its expression will bring much convenience to the following analysis.

Based on the power flow equations parameterized by μ, we introduce two common types of voltage instability, namely SNB and LIB. We take the P-μ nose curve of an arbitrary bus for illustration since all buses share a common μ and indicate the same stability limit point. Referring to a simple nose curve in Fig. 1, initially there exist a high-voltage solution and a low-voltage solution for each μ. As μ slowly increases, the two solutions get closer and coalesce at a nose point where no solutions exist if continue increasing μ. This point is known as an SNB point. Mathematically, the power flow Jacobian is singular at an SNB point as a simple zero eigenvalue emerges, while the power flow Jacobian is nonsingular elsewhere [1,15].

Next, we introduce the concept of LIB. Assume a generator hits its reactive power limit at a certain load level, then the corresponding bus losses voltage control capability and changes from a PV bus to a PQ bus. We refer to this point as a bus type switching (BTS) point, and the corresponding bus as the BTS bus. For example, two curves intersect at a BTS point in Fig. 2(a), where the blue[1] nose curve is the fixed-V curve which ignores the reactive power limit of the concerned generator, and the red nose curve is the fixed-Q curve where the reactive power generation is fixed to its limit value. At the BTS point in Fig. 2(a), the power flow solution is switched from the upper part of fixed-V curve to the lower part of fixed-Q curve. This BTS point is called an LIB point, which is regarded non-operable as it induces negative V-Q sensitivities and

[1] For interpretation of color in Fig. 2, the reader is referred to the web version of this article.