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Image despeckling with non-local total bounded variation regularization☆

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ABSTRACT

A non-local total bounded variational (TBV) regularization model is proposed for restoring images corrupted with data-correlated speckles and linear blurring artifacts. The energy functional of the model is derived using maximum a posteriori (MAP) estimate of the noise probability density function (PDF). The non-local total bounded variation prior regularizes the model while the data fidelity is derived using the MAP estimator of the noise PDF. The computational efficiency of the model is improved using a fast numerical scheme based on the Augmented Lagrange formulation. The proposed model is employed to restore ultrasound (US) and synthetic aperture radar (SAR) images, which are usually speckled and blurred. The numerical results are presented and compared. Furthermore, a detailed theoretical study of the model is performed in addition to the experimental analysis.

1. Introduction

Ultrasound is one of the widely used modalities in medical imaging due to its capability to provide images of moderately good quality without using the ionizing radiations. Moreover, the method is non-invasive in nature and causes relatively less harm to the subject. There are different models of ultrasound imaging, most of the conventional systems employ amplitude based or intensity based techniques for acquiring images. In amplitude based techniques the amplitude of the sound signal is recorded as a function of time and in intensity based techniques, the intensity is used in place of the amplitude. There are other ultrasound models as well, some of them use Doppler frequency information, see [1] for details.

Ultrasound signals transmitted by the systems undergo three major types of scattering depending on the characteristic of the object on which it is falling. The major types of scattering are specular, diffusive and diffractive. The specular scattering is due to the large size of the object compared to the wavelength of the signal and causes speckles in the captured data. Due to the presence of the speckles the signal intensity shoots up at the concerned pixel value, causing the signal to fluctuate the intensity values between a high range. It is shown in [2] that when the scatter density is more than 10; the speckle noise is found to follow Rayleigh distribution. However, the images (both satellite and medical) are formed adding different image slices whose intensity values follow a negative exponential law, the summed data is found to follow the Gamma law. In US and SAR imaging, the intensity of the resultant image is formulated as a product of reflectance of the sound and speckle i.e. ($I = RS$) therefore, the speckles are also found to follow the Gamma law.

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The remaining parts of the paper are arranged as follow. In Section 2 we deal with the basic concepts of image restoration under data-correlated speckle noise. Section 3 the novel contributions of the proposed work is highlighted. Section 4 designs the proposed framework for restoration of images corrupted with data-correlated speckle and linear blurring artefacts. Further, the numerical implementation of the model using the Augmented Lagrangian framework is being detailed in Section 4.1. We give detailed experimentation results in Section 5. The last section concludes the work throwing some lights on future expansion possibilities of the work. Furthermore, the theoretical analysis of the proposed solution is provided in Appendix A.

2. Image restoration under speckle noise

Since speckle formation is not an independent random process. The speckles are generally regarded as data-correlated noise. Moreover the speckles are multiplicative in nature (i.e) $u_0 = un$, (where $u \in \mathbb{R}^{N}$[1] and $u_0 \in \mathbb{R}^N$ are the original and noisy images, respectively) in the sense it gets multiplied with the intensity values making the data depend on it. This data-correlated nature of speckle makes its removal tedious.

There are several partial differential equations(PDE) and variational methods proposed in the literature to remove speckles from images [3,4]. These PDE models derive their concept from the non-linear diffusion model proposed by Perona–Malik for data-uncorrelated noise. These models reformulate the coefficient of diffusion in light of the well known statistical filters proposed by Kuan [5] and Lee [6]. Speckle reducing anisotropic diffusion [3] and its variants [4,7] use the coefficient of variation derived using Lee and Kuan filters in place of diffusion coefficient and archives the despeckling pretty well in US images. Nevertheless, all these second-order diffusion models tend to smooth-out the gradient oscillations along with the speckles and results in homogeneous intensity images; causing severe damages to the textures and details. As commonly observed, these second-order non-linear diffusion models perform linear approximations for non-linear functions; resulting in piecewise-linear stair formation, eventually making the output visually less appealing. In Li et al. [8], the authors device a curvature driven model for removing speckles in SAR images. Moreover, an ideal workaround to deal with multiplicative noise in general is the transformation of the intensities to the log domain, where the noise appears close in similarity to the data-uncorrelated Gaussian. Thereby making the process of despeckling literally similar to Gaussian denoising (for an additive noise model), see [9] for the details. However, it is worth noting that the transformation to the corresponding log domain does not make noise completely independent of the data as proved in [10]. In [11] the authors logarithmically transform the noise and the wavelet coefficients in each sub-band using heavy-tailed blur, assuming the noise as Gaussian. Recently a statistical similarity based approach is proposed for despeckling, this model is an improvement of the existing non-local means filter [12], where the similarity is evaluated based on statistical features, the details can be found in [13]. An enhanced Weiner filter is also proposed by the same authors for alleviating speckles in US data, see [14]. As already pointed out, assuming a data independence of noise makes the restoration weak due to the existing noise-signal correlation. Next we discuss some of the prominent variational formulations which duly consider the data-dependent nature of noise and linearity of the blurring operator.

Variational formulations started gearing-up ever-since Rudin et al. [15] framed their model to handle multiplicative (data-dependent) Gaussian noise. The first initiative in this direction for Gamma noise can be found in [10], where the authors formulate a functional based on the MAP estimate of the noise distribution (see Appendix A Section A.2 for the derivation of the MAP estimate). With the above assumptions the model is formulated as

$$\min_{u \in BV(\Omega)} J(u) = \min_{u} \left\{ \int_{\Omega} |\nabla u| dxdy + \lambda \int_{\Omega} \left(log(u) + \frac{u_0}{u} \right) dxdy \right\}. \tag{1}$$

Here $BV(\Omega)$ denotes the space of bounded variations (BV) and |.| denotes the usual L^1 norm (or absolute). Further x and y are the spatial variables. Due consideration of the devise related linear blurring artifact in the above model gives us

$$\min_{u \in BV(\Omega)} J(u) = \min_{u} \left\{ \int_{\Omega} |\nabla u| dxdy + \lambda \int_{\Omega} \left(log(Ku) + \frac{u_0}{(Ku)} \right) dxdy \right\}, \tag{2}$$

where $\Omega \subseteq \mathbb{R}^N$ is a bounded open subset with Lipschitz boundary, which is the area of image support, K is a linear bounded blurring operator, $\lambda > 0$ is the Lagrange multiplier, $\int |\nabla u|$ is the total variation (TV) of the functional u. As already mentioned earlier, this model also possesses all the drawbacks of the second-order non-linear diffusion model and further analysis reveals the fact that the second term (generally called as the data-fidelity) is conditionally convex giving raise to a non-unique solution. There are some improvements proposed for this model in terms of its restoration capability and theoretical stability, see [16,17] respectively, for the details. In Huang et al. [16] the authors have introduced a categorically convex data fidelity term which ensures the convexity invariably. In [17] the authors propose a Weberized TV model to improve the visual appearance of the filtered output. However, we note that a TV prior in the regularization term makes the results less appealing due to heavy penalization of gradient oscillations present in the textured data. Hereafter, we discuss a regularization prior which duly respects the local gradient fluctuations present in input images.

Preserving local image features is an important activity as far as most of the imaging modalities are concerned. As observed in many modalities, local image features play a vital role in characterizing the objects present in images. This

[1] For implementation purpose we columnize the images using column-stacking. Here N denotes size of the image i.e. $n \times m$.