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# Improvement of accuracy of the spectral element method for elastic wave computation using modified numerical integration operators

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## Abstract

We introduce new numerical integration operators which compose the mass and stiffness matrices of a modified spectral element method for simulation of elastic wave propagation. While these operators use the same quadrature nodes as does the original spectral element method, they are designed in order that their lower-order contributions to the numerical dispersion error cancel each other. As a result, the modified spectral element method yields two extra-orders of accuracy, and is comparable to the original method of one order higher. The theoretical results are confirmed by numerical dispersion analysis and examples of computation of waveforms using our operators. Replacing the ordinary operators by those proposed in this study could be a non-expensive solution to improve the accuracy.

*Keywords:* Elastic wave, FEM, SEM, Error-optimization, Numerical dispersion

## 1. Introduction

Finite element methods (FEMs) for computation of the elastic wave equation have greatly contributed to seismology and earthquake engineering [1–3]. Notably, the spectral element method (SEM) is most widely used in the past twenty years [4, 5]. For elastic wave computation, the SEM is usually associated with the Gauss-Lobatto-Legendre (GLL) quadrature rule and Lagrange polynomial basis defined on hexahedral elements, because this choice leads to an explicit time-marching scheme without loss of accuracy of computation. Detailed descriptions are available on [6, 7].

In applications of FEMs to elastic wave computation in complex underground structures, there may still exist difficulty concerning grid-generation. According to dispersion and stability analyses [8–14], it is preferable to use almost the same number of grid points per wavelength throughout the medium: i.e., in terms of accuracy, the number of grid points per wavelength should be sufficiently large to suppress numerical dispersion [10–14]; conversely, an unnecessarily large number of grid points (or small grid intervals) may increase the total number of time steps as well as computational cost required for each time step, since time intervals should be much smaller than the time for a wave train to pass through one grid interval [8, 9, 11]. In other words, we need a dense grid for a region of a lower propagation velocity, and a corse grid for a higher one, since the length of a wavelet depends on the propagation velocities. However, this condition makes the grid-generation more complicated as velocity structures become complex. Instead of regulating the number of grid points per wavelength, a regional increase/decrease of the order of elements would effectively improve the accuracy and efficiency. However, for the Legendre-type SEM (hereafter simply called the SEM), in particular, the non-equispaced distribution of the GLL nodes makes it difficult to connect elements of heterogeneous orders, without rather complicated implementations [15]. Moreover, a use of higher-order

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