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On the robustness of variational multiscale error estimators for the forward propagation of uncertainty

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Abstract

The numerical simulation of physical phenomena and engineering problems can be affected by numerical errors and various types of uncertainties. Characterizing the former in computational frameworks involving system parameter uncertainties becomes a key issue. In this work, we study the behavior of new variational multiscale (VMS) error estimators for the propagation of parametric uncertainties in a Convection-Diffusion-Reaction (CDR) problem. A sensitivity analysis is performed to assess the performance of the error estimator with respect to the mesh discretization and physical parameters (here, the viscosity value and advection velocity). Three different manufactured analytical solutions are considered as benchmarking tests. Next, the performance of the VMS error estimators is evaluated for the CDR problem with uncertain input parameters. For this purpose, two probabilistic models are constructed for the viscosity and advection direction, and the uncertainties are propagated using a polynomial chaos expansion approach. A convergence analysis is specifically carried out for different configurations, corresponding to regimes where the CDR operator is either smooth or non-smooth. An assessment of the proposed error estimator is finally conducted for the three tests, considering both the viscous- and convection-dominated regimes.

Keywords: Variational multiscale method; a posteriori error estimation; convection-diffusion-reaction equation; uncertainty propagation.

1. Introduction

The numerical simulation of physical phenomena has found widespread applications in the engineering sciences, and relies on approximating the state of a system with an approximate state, called numerical solution hereinafter. Let ϕ_{true} be a quantity of interest that represents a certain physical phenomena, e.g. the velocity field of a fluid. Let ϕ_{mod} be the quantity of interest given by the mathematical model that describes the physical phenomena. The numerical solution is computed on a certain mesh that discretizes a given domain with a certain characteristic size, h. We will denote as ϕ_h the numerical solution of the quantity of interest ϕ_{mod}. Following [1], we can distinguish between two types of error that are responsible of the difference between the true solution and the numerical solution (ε): the numerical error, ε_{num}, and the model error, ε_{mod}, related as

$$\varepsilon_{\text{num}} = \phi_{\text{mod}} - \phi_h, \tag{1a}$$

$$\varepsilon_{\text{mod}} = \phi_{\text{true}} - \phi_{\text{mod}}, \tag{1b}$$

$$\varepsilon = \phi_{\text{true}} - \phi_h = \varepsilon_{\text{mod}} + \varepsilon_{\text{num}}. \tag{1c}$$

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