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Cracking elements: A self-propagating Strong Discontinuity embedded Approach for quasi-brittle fracture

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A R T I C L E I N F O

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A B S T R A C T

In this paper, we present a *self-propagating* Strong Discontinuity embedded Approach (SDA) for quasi-brittle fracture. The method is based on the Statically Optimal Symmetric formulation (SOS) of the SDA using the 8-node quadrilateral element which avoids local stress locking. A non-continuous crack path is assumed such that the crack is modelled by a set of disconnected cracking segments. Hence, no complex crack tracking procedure and no explicit (or implicit) representation of the crack surface are needed. A local fracture criteria is proposed for determining the orientation of the crack. Several numerical tests with irregular discretizations are performed, demonstrating the effectiveness and robustness of the presented method.

1. Introduction

It is well known that damage in quasi-brittle materials is highly localized. Or in other words, the nonlinear softening behaviour leads to localization in a set of measure zero [1] and the width of the crack will quickly drop from finite size to almost zero during damaging process [2–5] leading to physically meaningless results in a numerical simulation. In last decades, many sophisticated models are developed for numerically simulating this strong discontinuity behaviour, such as classical interface element [6–8], remeshing techniques [9–13], multi-field mixed-mode formulation [14–19], models based on the screened Poisson equation [20,21], classic phase field models [22–31], numerical manifold method [32–36], meshfree methods [37–47], the cracking particles method [48–50] and even non-partial differential equations referred theory such as peridynamics [51–56]. Popular finite element methods without changing the initial discretization (re-meshing) include the eXtended Finite Element Method (XFEM) [57–63], the phantom node method [64–66] (as a special case of XFEM [67]) and Strong Discontinuity embedded Approach (SDA or EFEM) [68–81]. While XFEM introduces extra freedom degrees for describing the discontinuity field which needs to be solved for, the additional degrees of freedom in SDA can be condensed at element level and therefore require only minimum changes of an existing FEM code. Both methods show ignorable mesh dependence and robust numerical performance in fracture analysis of quasi-brittle materials. However, due to the lack of additional degrees of freedom, SDA captures the propagation process of cracks and mechanical response of the system more efficiently [82–85]. It was shown for several quasi-brittle benchmark problems in Ref. [86] that the numerical results obtained by SDA and XFEM are very similar.

Crack path continuity is commonly assumed for both XFEM and SDA, requiring a crack tracking strategy. Criteria predicting the location and orientation of a crack segment are based on the present stress, strain or energy state [73,76,87–91]. Some crack tracking strategies are capable of describing complicated even curved crack paths such as [73,92–95]. However, it is most common, to introduce straight crack segments once a crack propagates [67]. Crack tracking strategies can help to indicate the appropriate orientation of further crack opening and restrain the damage in local region, avoiding the so-called stress locking effect [96] caused by underestimation of softening, which many conventional smeared crack models commonly suffered from. Nevertheless, despite of the great advantage brought by tracking strategies, these methods are computationally expensive and complicated to implement, particular for complex crack patterns involving such as branching cracks. On the other hand, there are also SDA models with non-continuous cracks [96–101], bringing great flexibility for propagation of the cracks with less discretization dependency. However, it was reported that such approaches lead to stress-locking and too stiff responses [102] and in the worst case, the crack cannot propagate any more. Nonetheless, non-continuous crack paths allow for a self-

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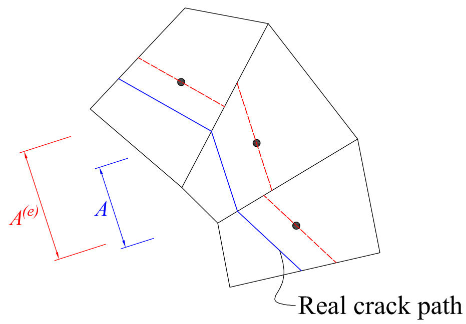

Fig. 1. Real length A of the crack used by standard SDA-SKON and equivalent length $A^{(e)}$ of the crack used by standard SDA-SOS.

propagating SDA in which the cracks are able to grow arbitrarily and automatically, not requiring specific crack tracking strategies.

There are 3 types of SDAs [102–105]: i) Kinematically Optimal Symmetric (SDA-KOS) formulation [102,106], ii) Statically and Kinematically Optimal Nonsymmetric (SDA-SKON) formulation [107–114], and iii) Statically Optimal Symmetric (SDA-SOS) formulation [115–118]. To the best of our knowledge, SDA-KOS is not very widely used. SDA-SKON is the most popular formulation, the standard version of which however needs the length as well as the orientation of the real crack. In other words, the standard version of SDA-SKON does not support non-continuous crack paths, as been pointed out in Refs. [104,119]. SDA-SOS, on the other hand, uses equivalent length of the crack, depending only on the orientation, see Fig. 1. This feature indicates SDA-SOS naturally supports non-continuous crack paths, making it a good candidate for developing self-propagating SDA.

In this work, we therefore present a self-propagating SDA method which does not suffer from just mentioned drawbacks. This is achieved by taking advantage of the higher order elements developed in our previous work [117] where we have proven that the stress locking problem of SDA-SOS can be solved by using higher order (quadratic) elements. The novelty and main features of our self-propagating SDA are summarized as follows:

1. The presented SDA inherits the advantages of the standard SDA-SOS presented in Ref. [117], i.e. (a) simple implementation, (b) good numerical stability, (c) no locking and no mesh-bias;
2. Unlike [117], this work uses noncontinuous crack path, which is self-propagating. Because no specific crack tracking strategy is used, the numerical procedures is greatly simplified and the computing effort for tracking (at least 10% of total computing time [117]) is greatly saved;
3. Unlike multi-crack SDA, this work uses single crack formulation in a single element, which is numerically much more stable. The direction of the crack changes with the loading process (non-fixed formulation) which might be comparable to rotating crack models.
4. Unlike classical rotating crack models presented in other publications in which the cracks could rotate 90° during loading, the rotating of cracks in this work are much slighter. More important, the benchmark tests indicate that our approach does not show the mesh dependences suffered by other rotating crack models.

The remaining parts of the paper are organized as follows: In Section 2, the formulation of the self-propagating SDA is presented, including the kinematics, the adopted mixed-mode softening law, iteration algorithm and fracture criteria. Numerical examples are presented in Section 3, including three point bending test, L-shaped panel test, a notched panel test and a dam model test. In the numerical tests, irregular discretizations are considered for evaluating the mesh-bias and robustness of the presented method. The paper closes with concluding remarks given in Section 4.

2. The formulation

2.1. Kinematics

Consider a domain Ω separated by a discontinuity $\mathbf{L}$ into Ω^+ and Ω^- with normal and parallel unit vectors $\mathbf{n}$ and $\mathbf{t}$, respectively. A localized subdomain Ω_φ is introduced leading to the displacement field of Ω

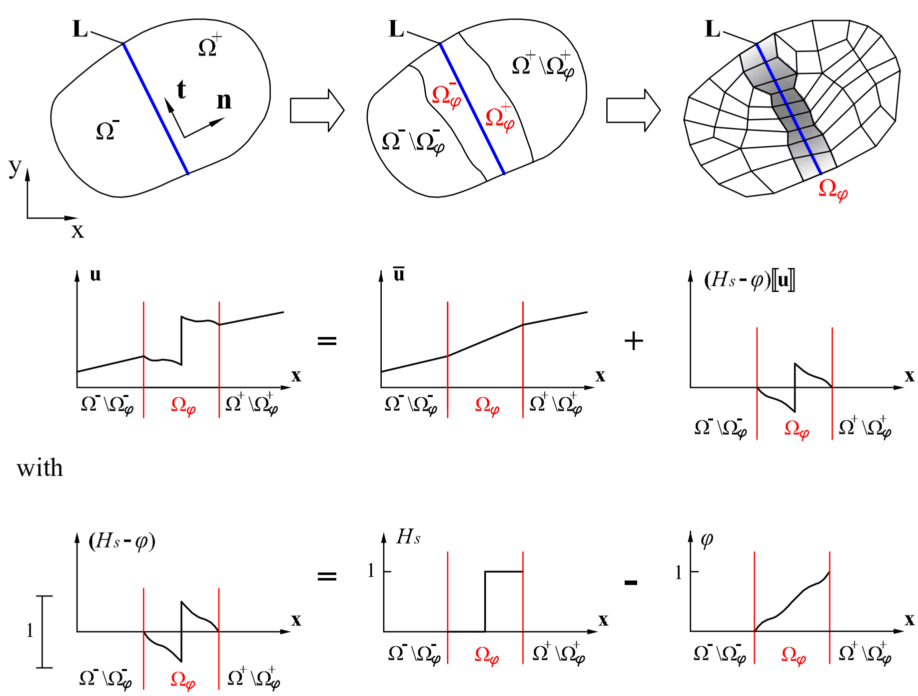

Fig. 2. Domain Ω and its displacement field.