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Pressure, temperature, and heat flux in high speed lubrication flows of pressurized gases

S.Y. Chien*, M.S. Cramer

Engineering Mechanics Program, Department of Biomedical Engineering and Mechanics, Virginia Polytechnic Institute and State University, Blacksburg, VA, 24060, USA

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ABSTRACT

We present approximate solutions to the compressible Reynolds equation and the corresponding temperature equation which are valid for large speed numbers in the dense and supercritical gas regime. The flows are taken to be two-dimensional, steady, compressible, single-phase and laminar. New results include explicit formulas for pressure, density, temperature, and heat flux in terms of the speed number, film thickness function, and the material functions. We have found that the first correction for finite speed number will depend on the local values of the effective bulk modulus and thermal expansion coefficient. Our approximations are compared to numerical solutions to the exact Reynolds theory. It was found that the first order approximation is necessary to obtain realistic pressure and temperature distributions.

1. Introduction

In many applications involving lubrication theory, the Reynolds equation plays a central role. Since first stated by Osborne Reynolds in 1886 [1], the Reynolds equation has been extended to include the effects of unsteadiness, turbulence, three-dimensionality, non-newtonian fluids and thermal effects [2–5]. While the conditions leading to the Reynolds approximation are frequently satisfied in many applications, see, e.g. Refs. [6–10], further motivation for studies of the Reynolds equation is that it provides valuable insights into more complex lubrication flows while in a relatively simple context.

Historically, large viscosity liquids are employed as lubricating fluids. In recent years there has been considerable interest in the use of both low and high pressure gases as working fluids [11–15]. The advantage of gases over large viscosity liquids include significant weight reduction, elimination of fouling and complications due to phase changes and the incompatibility with working fluids in power systems. Because the viscosities of gases tend to be smaller than those of liquid lubricants, lubricating gas flows require larger shear strains and are frequently compressible.

The theory of low pressure gas lubrication is well established in the literature where the perfect gas model is coupled with the Reynolds equation to account for compressibility effects [2–5]. The resulting Reynolds equation in these studies is typically cast as a nonlinear differential equation for pressure [2–5]. Both numerical and perturbation techniques are commonly employed to obtain solutions to the Reynolds equation. One of the first to derive perturbation solutions for low pressure gas films for high and low speed flows, i.e., large and small speed numbers (or bearing numbers) was Gross [5]. Peng and Khonsari [16] applied similar approach to estimate the lowest order hydrodynamic pressure for foil bearings with large speed numbers and ideal gases.

When the thermodynamic state is such that the lubricating fluids are no longer ideal, i.e., are in the dense or supercritical gas regimes, one must account for a strong dependence of material properties on the thermodynamic state and on rapid changes and singularities in the flow variables. In fact, even the validity of the Reynolds equation must be questioned in the supercritical gas regime, see, e.g., Chien, et al. [17,18]. Previous investigations such as [19–23] apply pure numerical schemes to different versions of the Reynolds equation. These studies account for the real-gas behavior of the lubricating gas through use of digital table look-ups. For example, studies [19–21] employed the NIST REFPROP database [24] and Guenat and Schiffmann [22] used the COOLPROP database [25]. Dousti and Allaire [23] have modeled the real gas behavior with a linear pressure-density relation, but this model is not expected to be valid over the full range of pressures and temperatures corresponding to the dense and supercritical regimes [26].

Because of the well known singularities in the supercritical gas regime, Chien, et al. [17] have carried out a detailed justification of the Reynolds equation. Limitations on the Reynolds equation were given. The corresponding simplified temperature equation was also derived. Even when the traditional thin film and lubrication approximations are

* Corresponding author.
E-mail address: armani23@vt.edu (S.Y. Chien).

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Nomenclature

β	Thermal expansivity (K^{-1})
ϵ	Eccentricity ratio
κ_{Te}	Effective bulk modulus (s^{-1})
κ_T	Bulk modulus (Pa)
Λ	Speed number
μ	Shear viscosity (kg $m^{-1}s^{-1}$)
ρ	Density (kg m^{-1})
c	Radial clearance (m)
c_p	Specific heat at constant pressure (kJ $kg^{-1}K^{-1}$)
Ec	Eckert number
h	Film thickness (m)
k	Thermal conductivity (W $m^{-1}K^{-1}$)
L	Length scale in the main flow direction (m)
p	Absolute pressure (Pa)
Pr	Prandtl number
q	Heat flux (W m^{-1})
r_f	Recovery factor
Re	Reynolds number
T	Temperature (K)
T_R	Temperature at the rotor (K)
T_S	Temperature at the stator (K)
U	Speed of the rotor surface (m s^{-1})
u	Non-dimensional x-component of velocity
V	Specific volume (m^3kg^{-1})
v_x	x-component of velocity (m s^{-1})
v_y	y-component of velocity (m s^{-1})

valid, it was shown that the Reynolds equation and its corresponding temperature equation break down simultaneously in the vicinity of the thermodynamic critical point. Solutions to Chien et al.'s [17,18] Reynolds and temperature equation were compared to solutions of the full Navier-Stokes equations revealing excellent agreement in the stated range of validity of the theory.

The goal of the present study is to develop explicit analytical solutions for density, pressure, temperature, and wall heat flux for high speed lubrication flows corresponding to a simple journal bearing. We follow the approach of [19–22] in that we analyze the Reynolds equation. In particular, we base our calculations on the Reynolds and temperature equation derived by Ref. [17]. We take the speed (or bearing) number to be large and present the first correction to the lowest order theory; here the term "lowest order" will typically refer to the approximation corresponding to an infinite speed number. The advantage of this work is that the dependence on speed number and thermodynamic state is explicit. The present work complements the extensive, but purely numerical, previous studies of pressurized gases [19–22] and the perturbation analysis of ideal gases by Gross et al. [5].

In the next section we describe the specific configuration and thermal boundary conditions to be considered. We take the flow to be sufficiently far from the near-critical regime so that Chien et al.'s [17] Reynolds equation and its corresponding simplified temperature equation can be regarded as valid. Exact solutions to Chien et al.'s [17] temperature equation are also presented in Section 2. In Section 3 we present the approximate solutions for density, pressure, temperature and heat flux valid for large speed numbers. In Section 4 we compare these approximate solutions to numerical solutions of the Reynolds equation and the corresponding temperature equation. The numerical solutions are generated using realistic and explicit models for the equation of state, viscosity, and thermal conductivity.

2. Formulation

We consider a two-dimensional flow in a thin gap corresponding to the configuration sketched in Fig. 1. This representation is a reasonable representation of a (two-dimensional) journal bearing if the clearance is small.

The flow contained in the region $0 \le x \le L$ and $0 \le y \le h(x)$ is taken to be steady, single-phase, compressible, and laminar. All physical variables are taken to have identical values at x = 0 and x = L. The film thickness h(x) is any sufficiently smooth function that satisfies

$$h(0) = h(L) \text{ and } \frac{dh}{dx}(0) = \frac{dh}{dx}(L) = 0. \quad (1)$$

A specific form of h(x) which corresponds to a two-dimensional journal bearing having a thin gap is provided in Section 4. Axial flow is not considered so that solutions are expected to be valid near the center plane of a long bearing. The upper surface, i.e., $y = h(x)$, is at rest and varies with the length scale L. The lower surface, i.e., $y = 0$, is translating with constant speed U in the positive x-direction. For convenience, we will refer to the upper and lower surfaces as the stator and rotor surfaces, respectively. Thus, the no-slip and kinematic boundary conditions require

$$v_x = U, v_y = 0 \; at \; y = 0, \quad (2)$$

$$v_x = v_y = 0 \; at \; y = h(x), \quad (3)$$

where v_x and v_y represent the velocity components in the x and y directions. Thermal boundary conditions include constant temperature walls where we specify both upper and lower surfaces with fixed temperatures. We will also consider boundary conditions corresponding to an adiabatic wall at either $y = h(x)$ or at $y = 0$ with a fixed known temperature at the non-adiabatic wall.

The Reynolds equation derived by Chien et al. [17] can be written in non-dimensional form as

$$\frac{d}{d\bar{x}}\left(\bar{h}^3\bar{\kappa}_{Te}\frac{d\bar{p}}{d\bar{x}}\right) = \Lambda\frac{d(\bar{\rho}\bar{h})}{d\bar{x}}, \quad (4)$$

where $\bar{x} = x/L$ and $\bar{h} = h(x)/h_0$; here $h_o \equiv$ h(0) and is the measure of the thickness of the fluid film. We take values of quantities evaluated at $\bar{x} = 0$ as the constant reference values and denote these values by the subscript "ref". We denote the fluid density by ρ so that $\bar{\rho} \equiv \rho/\rho_{ref}$ is the scaled density. The bulk modulus of the fluid is

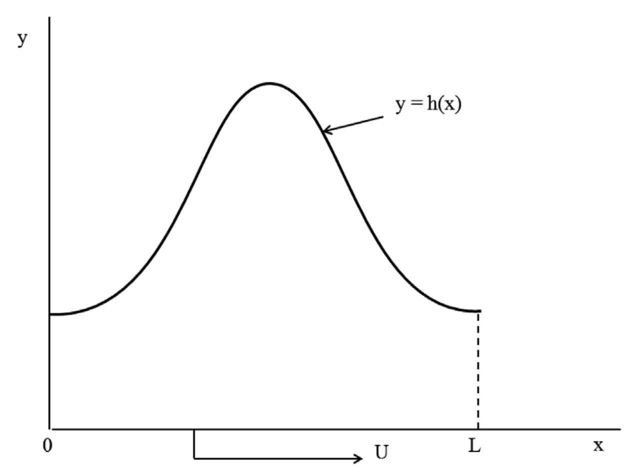

Fig. 1. Unwrapped configuration of a journal bearing. The y = 0 axis corresponds to the surface of the rotor and y = h(x) denotes the approximate position of the stator. The value of x is the distance measured along the rotor surface from the point of minimum film thickness. The quantity L denotes the circumference of the rotor and U denotes the constant speed of the surface of the rotor. The fluid is contained in the space 0 ≤ y ≤ h(x), 0 ≤ x ≤ L. Only x and y variations are considered and all velocity vectors will lie in the x-y plane.