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The distribution of cross sectional momentum returns[☆]

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Although there is a vast empirical literature on cross sectional momentum (CSM) returns, there are no known analytical results on their distributional properties due, in part, to the mathematical complexity associated with their determination. In this paper, we derive the density of CSM returns in analytic form, along with moments of all orders, under the assumption that underlying asset returns are multivariate normal. The resulting expressions are highly non-trivial in general and involve truncated normal distributions. The distribution of CSM returns can be formally described as a mixture of the unified skew-normal family of distributions. However, if the asset returns are independent, then the density of the CSM returns is shown to be a mixture of univariate normals. In order to shed light on the general case, we present a detailed analysis of the case of two underlying assets, which is shown to explain many of the key features of CSM returns reported in the empirical literature.

1. Introduction

Momentum based trading strategies rely on the persistence in the relative performances of assets over successive time periods called the ranking and holding periods respectively. For example, a cross sectional momentum (CSM) strategy, considered in Jegadeesh and Titman (1993), sorts the returns from n assets over the ranking period, and constructs a portfolio over the holding period consisting of an equally weighted long position in the m_+ best performing assets ("winners") and an equally weighted short position in the m_- worst performing assets ("losers"). Such strategies have become popular in practice, due mainly to the fact that they tend to exhibit positive returns. Another reason for their popularity is due to Carhart (1997), who pioneered momentum as a key component in the factor based risk analysis of investment returns.

Since it has been argued that excess returns from momentum based trading strategies would indicate a violation of the assumption of market efficiency, the returns generated by these strategies have been the subject of considerable empirical research spanning extensive asset classes, jurisdictions, and investment periods. Various authors, including Jegadeesh and Titman (1993, 2001), Asness (1994) and Israel and Moskowitz (2013), found that momentum strategies are profitable in US equities markets over different time periods dating back to 1927. Analogous results were found for country equity indices by Richards (1997), Asness et al. (1997), Chan et al. (2000) and Hameed and Yuanto (2002), for emerging markets by Rouwenhorst (1998), for exchange rate markets in Okunev and White (2003) and Menkhoff et al. (2012), for commodities by

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Erb and Harvey (2006), for futures contracts in Moskowitz et al. (2012), and in industries by Sefton and Scowcroft (2004). Similar results were also found by Asness et al. (2013) and Daniel and Moskowitz (2016) for markets in the European Union, Japan, United Kingdom and United States, and across asset classes including fixed income, commodities, foreign exchange and equity from 1972 through 2013.

In contrast to the extensive literature on the empirical properties of momentum based returns, there are relatively few that consider the distributional properties of these returns from a theoretical viewpoint, and none of these address the CSM returns as defined in this paper. Most of the known theoretical results, obtained for example by Lo and MacKinlay (1990), Jegadeesh and Titman (1993), Lewellen (2002) and Moskowitz et al. (2012), are concerned only with the expected values and the first order autocorrelations of the returns from the so-called weighted relative strength strategy in which the portfolio over the holding period is constructed from all underlying assets weighted, essentially, in proportion to their absolute or relative returns over the ranking period.

By assuming that underlying asset returns are Gaussian, we derive in this paper the distribution and the moments of CSM returns in the general case, and in a number of special cases under which resulting expressions simplify significantly. Anticipating our results, the densities obtained involve truncated normal distributions, which is a result partially discussed in Grundy and Martin (2001) Appendix A. The results generalize naturally to arbitrary fixed weight portfolios, albeit with added complexity in notation.

The remainder of this paper is organized as follows: Section 2 introduces the notation and the key results on multivariate normal distributions, and Section 3 provides a mathematically precise definition of CSM returns. Although the expressions for the CSM return density and moments are quite complex in general, they simplify considerably in the case of two assets with one winner and one loser, and this special case is examined in detail in Section 4. Implications of the results in the 2-asset case are considered in Section 5, where they are used to explain many of the empirical features reported in the literature. Numerical examples illustrating the different CSM return distributions that can be generated using parameters estimated from market data are given in Section 6, the distributional properties of CSM returns in the general case are derived in Section 7, and the paper concludes with Section 8.

2. Notation and preliminary discussion

For the convenience of the reader, we introduce in this section the notation that will be used throughout the paper, and present some preliminary discussion on cross sectional momentum returns to motivate the framework under which we develop the theory in later sections.

For any $\boldsymbol{x} \in \mathbb{R}^n$, we will write x_i for the i-th coordinate of $\boldsymbol{x}$, and given $\boldsymbol{y} \in \mathbb{R}^n$ write $\boldsymbol{x} \prec \boldsymbol{y}$ if and only if $x_i < y_i$ for all $1 \le i \le n$. Similarly, given a matrix $M \in \mathbb{R}^{m\times k}$, we will write $M_{i,j}$ for the (i, j)-th entry of M, and the transpose of a vector or a matrix will be denoted by the superscript $'$. The density of an n-dimensional normal distribution, with mean $\boldsymbol{\mu}$ and covariance Σ, at $\boldsymbol{x} \in \mathbb{R}^n$ will be denoted $\phi_n(\boldsymbol{x}; \boldsymbol{\mu}, \Sigma)$ so that

$$\phi_n(\boldsymbol{x}; \boldsymbol{\mu}, \Sigma) = \frac{1}{(2\pi)^{\frac{n}{2}} |\Sigma|^{\frac{1}{2}}} e^{-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu})'\Sigma^{-1}(\boldsymbol{x}-\boldsymbol{\mu})}, \tag{1}$$

and for any $\boldsymbol{a} \in \mathbb{R}^n$ we define

$$\Phi_n[\boldsymbol{a}; \boldsymbol{\mu}, \Sigma] = \int_{\boldsymbol{x}\prec \boldsymbol{a}} \phi_n(\boldsymbol{x}; \boldsymbol{\mu}, \Sigma)\, d\boldsymbol{x}, \tag{2}$$

where $d\boldsymbol{x} = dx_1, \dots, dx_n$. In general, given random variables $\boldsymbol{x}_1, \dots, \boldsymbol{x}_n$, their joint probability density function will be denoted $f_{\boldsymbol{x}_1,\dots,\boldsymbol{x}_n}$.

For any $n \in \mathbb{N}$, let S_n be the group of permutations of $\{1, 2, \dots, n\}$, and given any $\tau \in S_n$, denote by $\tau_i = \tau(i)$ the image of $1 \le i \le n$ under τ, so that, for example,

$$S_3 = \{(1\,2\,3), (1\,3\,2), (2\,1\,3), (2\,3\,1), (3\,1\,2), (3\,2\,1)\}. \tag{3}$$

We now provide a brief description of CSM returns and some explanation of the relevance of our assumptions. Let $l_r \in \mathbb{N}$ be the length of the ranking period and $l_h \in \mathbb{N}$ the length of the holding period. Then the construction of a cross sectional momentum strategy can be described as follows. Firstly, in month t, all underlying assets are ranked and sorted into quantiles, for example quintiles or deciles, based on their returns over the past l_r-month ranking period from month $t - l_r - 1$ to $t - 1$. One month is omitted to avoid short-term reversals and similar phenomena. At this time, the top quantile portfolio, consisting of “winners”, is purchased and the worst performing quantile, consisting of “losers”, is shorted for the l_h-month holding period from month t to $t + l_h$. The two portfolios may be equally weighted, value weighted, or more generally weighted arbitrarily using fixed weights.

The problem of determining the distributional properties of the CSM returns can be regarded as a 2-period problem. At any time t, the first period is the ranking period from $t - l_r - 1$ to $t - 1$ over which the asset returns are described by the vector $\boldsymbol{r}_t$, and the second period is the holding period from $t + 1$ to $t + l_h$ over which the corresponding asset returns are $\boldsymbol{r}_{t+1}$. The vector $(\boldsymbol{r}'_t, \boldsymbol{r}'_{t+1})$ is assumed to be Gaussian, and since we do not make any assumptions on stationarity the joint distribution of $\boldsymbol{r}_t$ and $\boldsymbol{r}_{t+1}$ is a $2n$-dimensional vector of normal returns with means and variances depending on t and $t + 1$ as in Assumption 1. This flexibility means that the exclusion of the one month period from $t - 1$ to t or the assumption $l_r \ne l_h$, for example, do not pose any difficulties.

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