Journal of Mathematical Economics 78 (2018) 52–58

Contents lists available at ScienceDirect

Journal of Mathematical Economics

journal homepage: www.elsevier.com/locate/jmateco

Strategic vote trading under complete information

Dimitrios Xefteris, Nicholas Ziros *

Department of Economics, University of Cyprus, P.O. Box 20537, Nicosia 1678, Cyprus

ARTICLE INFO

Article history:
Received 14 December 2017
Received in revised form 14 July 2018
Accepted 31 July 2018
Available online 9 August 2018

Keywords:
Vote trading
Complete information
Strategic market games
Power sharing
Proportional justice

ABSTRACT

We study two-party elections considering that: (a) prior to the voting stage voters are free to trade votes for money according to the rules of the Shapley–Shubik strategic market games; and (b) voters' preferences – both ordinal rankings and cardinal intensities – are public information. While under plurality rule no trade occurs, under a power-sharing system (voters' utilities are proportionally increasing in the vote share of their favorite party) *full trade is always an equilibrium* (two voters – the strongest supporter of each party – buy the votes of all others). Notably, this equilibrium implements *proportional justice* with respect to the two buyers: the ratio of the parties' vote shares is equal to the ratio of the preference intensities of the two most opposing voters.

1. Introduction

Vote markets have attracted the interest of scholars from many disciplines because they allow voters to express their preference intensities over alternatives; not only their ordinal preferences as simple voting. Despite the fact that both complete and incomplete information environments regarding the voters' preferences are relevant in different real life applications,[1] most works in the literature feature incomplete information. Indeed, when voters' preferences are their private information vote-trading equilibria have been shown to exist and have been characterized in a variety of contexts (see, for instance Casella et al., 2012, 2014; Casella and Turban, 2014; Xefteris and Ziros, 2017), while much less is known about the case in which voters' preferences are public information.

In this paper, we try to fill this gap by studying the consequences of vote trading in the least explored informational environment of complete information. Undeniably, incomplete information is a reasonable assumption in many cases, but a proper understanding of vote markets – as it is the case with markets for standard goods – may be achieved only if we have a good idea of what to expect in a complete information setting as well. Our approach employs a strategic rather than a price-taking exchange framework, as vote trading is conducted via the mechanism of strategic market games (introduced in Shubik, 1973; Shapley and Shubik, 1977), which maps agents' actions to prices and allocations. We study a non-cooperative game in strategic form which allows us (a) to use Nash equilibrium as a solution concept and (b) not to impose any price-taking hypothesis as the standard approaches on vote markets (e.g., Piketty, 1994; Philipson and Snyder, 1996; Casella et al., 2012); and hence to effectively deal with the conceptual and practical problems of competitive equilibrium analysis in markets with externalities (see, for instance del Mercato, 2006).[2] In particular, we study a two-party election in which prior to the voting stage individuals are free to trade votes for money, if they find it profitable to do so. That is, an individual can offer her vote in exchange for money or can place a monetary bid in exchange for votes. Hence, the price of a vote is endogenously determined by the actions of vote traders, while the distribution mechanism allocates the supplied votes to vote buyers in proportion to their bids and accordingly distributes monetary bids to those who chose to sell their votes.

In this framework we investigate the effect of vote trading with complete information under different electoral systems. Initially, we briefly argue that under the simple plurality rule the unique equilibrium involves all players abstaining from vote trading. This is perfectly in line with earlier approaches (e.g. Piketty, 1994; Casella et al., 2012; Casella and Turban, 2014) which also conclude that under plurality rule and complete information, we cannot expect stable vote trading to take place before the elections. Then, we move on to examine whether an alternative electoral system, which avoids some deficiencies associated with plurality rule (e.g., severe discontinuities in the outcome function), can guarantee a generic existence of an equilibrium with vote trading. To this

* Corresponding author.
E-mail addresses: xefteris.dimitrios@ucy.ac.cy (D. Xefteris), n.ziros@ucy.ac.cy (N. Ziros).

[1] For instance, in most legislatures or committees it is not realistic to assume uncertainty about the policy preferences of their members.

[2] Employing the rules of strategic market games is not the only way to evade such complications. Another solution is to modify the competitive equilibrium notion so that it properly deals with the particularities of vote trading (see, for instance Casella et al., 2012, 2014; Casella and Turban, 2014).

https://doi.org/10.1016/j.jmateco.2018.07.009
0304-4068/

end, we consider a power-sharing system, in which the decision-making power is distributed between the two competing parties in proportion to their vote shares. Similar frameworks have been extensively employed in the political economics literature,[3] but, to the best of our knowledge, only Xefteris and Ziros (2017) have studied vote trading in such systems.[4] In such a setup the whole distribution of votes is crucial for the determination of policies and a voter's utility is proportionally increasing in the vote share of her favorite party.

We provide a full characterization of all Nash equilibria under the power-sharing electoral rule. Apart from the no-trade equilibrium we show that, for every generic preference profile, there exists a unique full-trade equilibrium. In this equilibrium only two players, the strongest supporter of each party, are buying votes whereas all the other players prefer to sell their votes. Moreover, we show that partial-trade equilibria might exist, but only for specific classes of preference profiles. That is, depending on the precise preference profile we might additionally have equilibria in which trade occurs, but not among all players. In these equilibria, again, only the strongest supporter of each party buys votes, some players sell their votes while the rest – with preference intensities within a party-specific interval – prefer to refrain from vote trading and simply vote for their preferred party during the elections. Hence, in all equilibria with active trading the competition between two vote buyers determines in a large degree the final vote shares of the two parties. It should be noted that similar results with respect to the number of vote traders have been obtained by the means of alternative equilibrium concepts and institutional settings in Casella et al. (2012, 2014) and Casella and Turban (2014), where two voters demand votes and all other voters offer their votes for sale.[5]

Concerning the welfare properties of vote trading, the earlier literature has produced both positive (for example Buchanan and Tullock, 1962) and negative (for example Riker and Brams, 1973) results about the superiority of vote trading over the no-trade option, focusing on Benthamite/utilitarian criteria. More recently, Casella and Turban (2014) showed that vote trading is welfare decreasing when compared to plurality rule without vote trading, in the sense that the minority's favorite policy is implemented with higher probability than the efficient level. On the other hand, Xefteris and Ziros (2017), in an incomplete information variant of the current framework, proved that vote trading is welfare improving because when vote trading is allowed all players' expected utility is larger compared to the case where vote trading is prohibited. The welfare analysis of vote trading under complete information exhibits that both welfare improving and decreasing outcomes are possible, since in certain cases vote trading leads to a larger social utility compared to simple voting, and in some others not.

In fact, vote trading under complete information in power-sharing systems is found to implement an alternative social choice rule: it achieves *proportional justice in policy* with respect to the two buyers. That is, the ratio of the parties' vote shares is equal to the ratio of the preference intensities of their strongest supporters. The origins of proportional justice with respect to a distributional problem involving two individuals may be traced back to Aristotle and it has been recently studied by Broome (1984, 1991) and Segal (2006), in a more standard economics' context.[6] This result is arguably of independent interest as, to the best of our knowledge, this is the first known mechanism that takes into account only each party's stronger supporter and is "fairly biased" – in the context of Segal (2006) – towards the one with the most intense preferences. Of course this welfare analysis holds specifically for our unique full-trade equilibrium, and does not extend to other outcomes possibilities. But since in our complete information environment, the full-trade equilibrium is the unique one that exists for every generic preference profile, it is the only reasonable candidate for a comprehensive welfare analysis: other equilibria might only deliver insights for merely a fraction of possible preference distributions. It should be stressed here that – as it is also argued by Casella and Turban (2014) – social choices that take in account the preferences of only two individuals are highly undemocratic and hence not particularly appealing. However, from a neutral implementation-theory perspective, it is arguably quite interesting to identify mechanisms that implement any well-defined social choice rule.

Overall, our analysis underlines the importance of three interacting aspects of the vote-trading environment: (a) the information that voters' hold about their fellow citizens' preferences — as equilibria under plurality rule may exist with incomplete information and not with complete information, (b) the voting rule — as under complete information equilibria with active vote-trading exist under a power-sharing rule and not under plurality rule, and (c) the vote-trading mechanism in operation — as the standard Walrasian setup cannot properly nest the externalities involved in vote trading. It is shown that under purely strategic vote-trading institutions, equilibria exist under both plurality and power-sharing rules, even with the demanding assumption of complete information. Indeed, in the first case (plurality rule) the only equilibrium outcome is that no trade takes place and in the second case (power sharing), in the most robust equilibrium of the game, all voters engage in vote trading. But in both cases equilibrium behavior is well-defined and possible to be fully characterized in a unified framework.

The remainder of the paper is organized as follows. In Section 2 we develop the model, in Section 3 we present the main results and in Section 4 we discuss the welfare properties of the full-trade equilibrium. Some concluding remarks follow in Section 5. The discussion about partial-trade equilibria can be found in the Appendix.

2. The model

We consider a committee of $n > 2$ voters and two parties (or policy alternatives), L and R. Voters fall into two types depending on their ordinal preferences, $t_i \in \{L, R\}$, where $t_i = L$ if $L \succ R$ and $t_i = R$ if $R \succ L$ for voter i. Hence we have two sets of voters with cardinality $n_L \geq 1$ and $n_R \geq 1$, respectively, where $n_L + n_R = n$. Each voter i is also characterized by her distinct intensity parameter $w_i > 0$ and let us denote with $\bar{w}^L$, $\bar{w}^R$ the valuations of the each party's strongest supporter. All voters have one vote each and concerning their monetary endowments we

[3] See, for instance, Lijphart (1984), Austen-Smith and Banks (1988), Ortuño Ortín (1997), Alesina and Rosenthal (2000), Llavador (2006), Sahuguet and Persico (2006), Herrera et al. (2014), Iaryczower and Mattozzi (2013), Saporiti (2014), Matakos et al. (2016) among others.

[4] That paper considered incomplete information regarding voters' preferences – which, as explained above, enhances the prospects of equilibrium existence –, symmetric uncertainty – in the sense that no party is expected to be supported by a more voters than the other – and restricted strategy spaces — in the sense that vote buyers were not allowed to bid any arbitrary monetary amount. In this paper, we consider instead complete information, arbitrary voters' preferences and unrestricted strategy spaces: each voter is free to bid *any monetary amount* she deems best. Hence, this paper is essentially *the first one* that employs a complete strategic market game framework in a vote-trading model.

[5] In Casella et al. (2012) the two vote buyers are those with the overall highest intensities (irrespectively of their party preferences), whereas in Casella et al. (2014) and Casella and Turban (2014) only the strongest supporter of each party buys votes.

[6] We should note that we consider implementation in the limit: we demonstrate that when the number of voters becomes arbitrarily large, the full-trade equilibrium is such that the ratio of the vote shares of the two parties converges to the ratio of the preference intensities of the two buyers.