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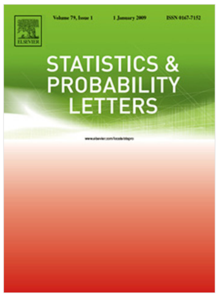

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Optimal directional statistic for general regression

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Abstract

For a general linear regression model we construct a directional statistic which maximizes the probability that the scalar product between the vector of unknown parameters and any linear estimator is positive. Special emphasis is given to comparison of this directional statistic with the BLUE and explaining why the BLUE could be relatively poor. We illustrate our results on analytical and numerical examples.

Keywords: Regression; Optimal direction; BLUE; Correlated errors.

1 Introduction and formulation of the main result

Consider the general linear regression model

$$y(x) = \theta^T f(x) + \varepsilon(x), \;\; x \in \mathcal{X}\,, \tag{1}$$

where $\mathcal{X}$ is a bounded Borel subset of $\mathbb{R}^d$ with $d \geq 1$, $\theta = (\theta_1, \ldots, \theta_m)^T$ is a vector of unknown parameters, $f = (f_1, \ldots, f_m)^T$ is a vector of base functions and $\varepsilon(x)$ is a Gaussian random noise process (or field) with zero mean and finite covariances $\mathbb{E}\varepsilon(x)\varepsilon(x') = \sigma^2 K(x, x')$ for $x, x' \in \mathcal{X}$, where $\sigma^2 > 0$ may be unknown and $K(\cdot, \cdot)$ is a known positive definite function (kernel) on $\mathcal{X} \times \mathcal{X}$. By Ξ_1 and Ξ_m we denote the linear spaces of all finite signed measures defined on (Borel subsets of) $\mathcal{X}$ and all signed m-vector measures on $\mathcal{X}$, respectively (a signed measure $\nu \in \Xi_1$ can be thought of as a difference