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Research Paper

Application of different curve interpolation and fitting methods in water distribution calculation of mobile sprinkler machine

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In the calculation of water application in mobile sprinkler machines, various interpolation and fitting methods have been applied to describe the radial water distribution pattern of sprinklers. However, few studies have been carried out related to the accuracy and applicability of these methods. In this study, the radial water distribution curves were obtained by adopting cubic spline interpolation, Lagrange interpolation, least-squares polynomial fitting, and simplified geometric curves, respectively. The mobile spraying water distribution and spray uniformity coefficients were calculated based on these radial water distribution curves. The results indicated that the cubic spline curve and the least-squares polynomial fitting reflected the water distribution characteristics accurately, with the maximum deviation between the calculated values and the measured values being less than 10%. The existence of Runge's phenomenon in the Lagrange interpolation method led to sharp oscillations at both ends of the radial water distribution curve, resulting in large inconsistencies between the calculated value and the measured value of irrigation depth. The curves of the simplified geometric method were too simple to accurately characterise water application. Although the cubic spline interpolation showed high calculation accuracy, the large number of interpolation polynomial coefficients hinder its practical application. The least-squares based polynomial fitting curve showed both sufficient calculation accuracy and simple expression form, while the number of polynomial coefficients is only one more than the degree of polynomial. The recommended degree of polynomial fitting was six in this study.

1. Introduction

Irrigation uniformity is an important indicator in quantifying irrigation performance (Burt et al., 1997). Hard hose travellers produce a spray with a low level of uniformity when compared to other sprinkler systems (Keller & Bliesner, 1990). Wigginton and Raine (2001) tested and determined the distribution uniformity coefficient of hard hose travellers in Mary Valley. The test results of distribution uniformity coefficient

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ranged from 1% to 88%, with an average value of 62%. There are generally two methods to improve upon the spray uniformity. One method is the investigation of the system configuration and operation parameters' influence on water distribution through a field test, as in the work performed by Hashim et al. (2016) and Jangra (2017). An advantage of a field test is the exact reproduction of field water distribution, while a disadvantage is that this method is resource intensive with regards to labour and equipment. Another method to improve upon spray uniformity is to use irrigation calculation software, such as DEPIVOT (Valín et al., 2012), SIRIAS (Carrión, Tarjuelo, & Montero, 2001), TRAVGUN (Smith, Gillies, Newell, & Foley, 2008) and ENROLADOR (Rolim & Teixeira, 2016), to simulate the spray distribution in the field. The common feature of the above software is that the calculation of the field water distribution is based on the radial water distribution curve of a fixed sprinkler. The radial water distribution curve, however, is not generally provided in the manufacturer's literature, which requires the researchers to establish some data through measurement. For instance, the software SIRIAS calls for the test data of Tarjuelo, Gómez, Pardo (1992), Tarjuelo et al. (1999). During the sprinkler radial leg test, catch cans are generally used to collect the applied water (Faci et al., 2001), however the number of catch cans is limited, which requires mathematical methods to obtain the estimated applied water between the catch can locations. Richards and Weatherhead (1993) adopted a cubic polynomial to predict the spray intensity of each point along the spray in the radial direction. Smith et al. (2008) adopted cubic spline interpolation to describe the radial water distribution of a spray nozzle and applied this method to the sprinkler calculation software TRAVGUN (Smith, Foley, & Newell, 2003). It was found that the distribution of the radial spray pattern followed a consistent shape (Prado et al., 2012; Rolim & Pereira, 2005, pp. 166–171), and because of this, some researchers described the radial water distribution of sprinklers as geometric shapes to simplify the calculation process. For instance, Rolim and Pereira (2005, pp. 166–171) used a triangular shape to simulate the radial water distribution of sprinklers. Prado et al. (2012) described the radial water distribution of sprinklers with triangular, elliptical and rectangular shapes, respectively, and the operating parameters of hard hose travellers were further discussed based on the three radial water application patterns. Kincaid (2005) also divided the water distribution of sprinklers into triangular, trapezoidal, and rectangular distributions, according to the ratio between maximum precipitation depth and average precipitation depth. The calculation produced by the DEPIVOT software was based on an assumption of an elliptical pattern of radial water distribution (Valín et al., 2012). It is apparent from the above research that a variety of methods, such as interpolation curves, polynomial fitting, and geometric shapes, have been adopted to describe the radial application patterns of sprinklers. These different forms of radial water distribution patterns have been further used by the calculation models to assess the water distribution and spray uniformity of the entire spray area. However, the calculated results of different radial water distribution curves would inevitably be different and no effort has been made to verify the rationality of each expression form and the accuracy of the corresponding calculated results. In this study, we would like to select an accurate and convenient radial water distribution expression from a variety of methods. The accuracy of model prediction under different radial water distribution curves was compared and quantitatively analysed by means of model calculations and actual measurements. The convenience of application was then discussed for the methods with high accuracy. Finally, the recommended curve form was used to improve the model calculation accuracy of mobile sprinkler water distribution.

2. Materials and methods

2.1. *Interpolation and fitting methods*

It is assumed that the actual radial water distributions of sprinklers satisfy the function $f(x)$ and the expression of the function is unknown. A set of points that satisfied the function $f(x)$ was obtained through the radial water distribution test. It is desirable to obtain a smooth curve $g(x)$ from the obtained set of points that is nearest the original curve function $f(x)$. The most commonly used methods to obtain $g(x)$ are the interpolation curve method and the fitting curve method. The two methods differ in that the interpolation curve method requires that the resulting curve pass through each given point, while the fitting curve method requires that the resulting curve be close to each given point, but not necessarily pass through each point.

2.1.1. *Cubic spline interpolation*

The cubic spline interpolation is composed of a set of cubic polynomials. The interpolation interval is divided into small intervals by node x_i ($a = x_0 < x_1 < x_2 < \ldots < x_n = b$). There is a cubic polynomial function in each interval $[x_{i-1}, x_i]$. The second order of each function is continuous, so that the cubic spline line curve is smooth and each segment connects with each other. Smith et al. (2008) showed that the radial water distribution of a sprinkler nozzle could be described by a cubic spline curve using Eq. (1). When the fitting coefficients are known, the expression of cubic spline of this segment is calculated.

$$P(x) = \frac{360}{\theta}\left[\frac{(x-x_1)^3}{6\delta x}w_2 + \frac{(x_2-x)}{6\delta x}w_1 + \left(\frac{P_2}{\delta x} - \frac{w_2\delta x}{6}\right)(x-x_1) + \left(\frac{P_1}{\delta r} - \frac{w_1\delta r}{6}\right)(x_2-x)\right] \quad (1)$$

For each segment of the cubic spline interpolation curve, $P(x)$ is the average application intensity at a distance of x from the sprinkler, mm h^{-1}, $x_1 < x < x_2$; P_1 and P_2 are the average application intensities at distances x_1 (point 1) and x_2 (point 2) from the sprinkler, mm h^{-1}, points 1 and 2 move for each segment; δ_r is the distance between point 1 and point 2, m; w_1 and w_2 are the fitting coefficients; θ is the rotating sector angle of the sprinkler, in degrees.

2.1.2. *Lagrange interpolation*

Since a set of points that satisfies $f(x)$ is available, we assume that the number of points is $n+1$ and construct a n-degree