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State feedback linearization of nonlinear control systems on homogeneous time scales

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ABSTRACT

The paper addresses the state feedback linearization problem for nonlinear systems, defined on homogeneous time scale. Necessary and sufficient solvability conditions are given within the algebraic framework of differential one-forms. The conditions concerning the exact dynamic state feedback linearization are equivalent to the property of differential flatness of the system. An output function which defines a right invertible system without zero-dynamics is shown to exist if and only if the basis of some space of one-forms can be transformed, via polynomial matrix operator over the field of meromorphic functions, into a system of exact one-forms. The results extend the corresponding results for the continuous-time case.

1. Introduction

The main goal of the paper is to extend the results of [1] on static state feedback linearizability, allowing dynamic state feedback. This problem will be studied for nonlinear systems, described on homogeneous time scales. Such systems incorporate continuous- and discrete-time systems as special cases. However, the discrete-time systems in the time scale formalism are described in terms of the difference operator, unlike the shift operator as in [2]. The reason is that the shift operator is not a special case of delta derivative, whereas the difference is. The theory of nonlinear systems on time scales dates back to [3]. Note that analysis on time scales is nowadays recognized as a proper tool to unify the study of continuous- and discrete-time systems [4,5]. The unification simplifies software development and allows in practice to analyze the system and its sampled-data model jointly. Moreover, the unification allows to see the differences between the discrete- and continuous-time cases or to prove that there are none. Likewise, besides unification, time scales calculus provides mathematical tools for extension on the arbitrary time domain. For example, some control problems for systems defined on arbitrary time scales were studied in [6–10]. In this paper, we, however, restrict our attention only to homogeneous time scales, since the extension from homogeneous to non-homogeneous is not trivial. Compared to the homogeneous case, the operators Δ_f and σ_f do not commute, but this difficulty is more of a technical nature, making the computations more complex. The main source of difficulty is that the additional time variable t appears in the definition of the differential ring. The latter requires that the new variables of the inversive closure, depending on t, have to be chosen to be smooth at each dense point t of the time scale, see [11]. Then one can construct the differential ring of meromorphic functions associated with the nonlinear control system. This differential ring is commutative, but it can have zero divisors, so it is impossible

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to construct the quotient field of this ring. Consequently, modules over this differential ring can be constructed instead of spaces over the field, see [11] and one gets much more complicated algebraic formalism. However, for some systems defined on non-homogeneous time scales the extension is still possible. One important case for control is non-uniformly sampled systems. Though the technical difficulties related to non-commutativity as described above show up, the smoothness issue does not since the time scale is purely discrete.

The solution of dynamic state feedback linearization problem relies on the so-called linearizing one-forms that were constructed in [1]. If these one-forms are exact then the linearization can be achieved via static state feedback. The dynamic state feedback solution can be understood as integrability aspect, and it builds a link between (not necessarily exact) linearizing one-forms and linearizing outputs. The linearizing outputs are output functions with respect to which the system is (right) invertible with no zero dynamics. Since invertibility conditions, necessary to address our main goal, have not been studied earlier for control systems that are defined on time scales, we have first to fill this gap. We give necessary and sufficient invertibility conditions together with the construction of the bases of spaces $\mathbb{E}_k$, $k \geq 1$, (defined in Section 4) that allows to check the above conditions. Moreover, we extend the concept of infinite zero structure for this class of systems. The latter allows easily to describe the subclass of systems without zero dynamics.

The algebraic framework of differential one-forms is used together with the theory of non-commutative polynomial rings built upon it. The polynomial indeterminate is interpreted as a delta derivative and polynomials act as operators on one-forms. In the continuous-time case the delta derivative is just an ordinary time derivative and our results recover those of [12].

The paper is organized as follows. Section 2 recalls the basic definitions of time scale calculus. Section 3 gives a brief overview of algebraic framework of differential one-forms. Invertibility conditions are given in Section 4. Since the proofs of the statements concerning the invertibility are similar to the results given in [2], we present them in Appendix A for the completeness of the paper and for showing that in the discrete time case we use the difference operator, while in [2] the shift operator is used. Dynamic feedback linearization problem is studied in Section 5. Section 6 is devoted to showing illustrative examples that describe our results. Concluding remarks are drawn in Section 7.

2. Preliminaries

2.1. Time scale calculus

For the introduction to the calculus on time scales, see [4,5]. Here, we recall only those notions and facts that will be used later.

A *time scale* $\mathbb{T}$ is an arbitrary nonempty closed subset of the set $\mathbb{R}$ of real numbers. For $t \in \mathbb{T}$ the *forward* and *backward jump operators* $\sigma, \rho : \mathbb{T} \to \mathbb{T}$ are defined by $\sigma(t) = \inf\{s \in \mathbb{T} | s > t\}$, and $\rho(t) = \sup\{s \in \mathbb{T} | s < t\}$, respectively. In addition, we set $\sigma(\max \mathbb{T}) = \max \mathbb{T}$ if there exists a finite $\max \mathbb{T}$, and $\rho(\min \mathbb{T}) = \min \mathbb{T}$ if there exists a finite $\min \mathbb{T}$. Then the *graininess* functions $\mu, \nu : \mathbb{T} \to [0, \infty)$ are defined by $\mu(t) = \sigma(t) - t$ and $\nu(t) = t - \rho(t)$, respectively, for all $t \in \mathbb{T}$. A time scale is called *homogeneous* if μ and ν are constant functions.

From now, we assume that $\mathbb{T}$ is a homogeneous time scale. The definition and the properties of the *delta derivative* f^{Δ} of a real function f can be found, for instance, in [4,5] and they are also recalled in [3,13].

Remark 1. The delta derivative is a natural extension of time derivative in the continuous-time case and respectively, forward difference operator in the discrete-time case. Therefore, for $\mathbb{T} = \mathbb{R}$, $f^{\Delta}(t) = \lim_{s\to t} \frac{f(t)-f(s)}{t-s} = f'(t)$ and for $\mathbb{T} = \mathbb{Z}$, $f^{\Delta}(t) = \frac{f(\sigma(t))-f(t)}{\mu(t)} = f(t+1) - f(t) =: \triangle f(t)$, where $\triangle$ is the usual forward difference operator.

Let $f^{[0]} := f$ and $f^{[1]} = f^{\Delta}$. For a function $f : \mathbb{T} \to \mathbb{R}$ we define higher order delta derivatives by $f^{[2]} : \mathbb{T} \to \mathbb{R}$, $f^{[2]} := \left(f^{\Delta}\right)^{\Delta}$ and $f^{[n]} : \mathbb{T} \to \mathbb{R}$, $f^{[n]} := \left(f^{[n-1]}\right)^{\Delta}$, for $n \geq 3$. Note that for a homogeneous time scale $f^{[n]}$, $n \geq 1$ are uniquely defined for all $t \in \mathbb{T}$.

3. Linear algebraic framework

Recall some definitions and facts from [3,13] that will be used in the paper.

Consider now the control system, defined on the homogeneous time scale $\mathbb{T}$,

$$x^{[1]}(t) = f(x(t), u(t)) \tag{1a}$$

$$y(t) = h(x(t)), \tag{1b}$$

where $(x(t), u(t)) \in X \times \mathcal{U}$, $X \times \mathcal{U}$ is an open subset of $\mathbb{R}^n \times \mathbb{R}^m$, $m \leq n$, and $f : X \times \mathcal{U} \to \mathbb{R}^n$, $h : X \to Y \subset \mathbb{R}^p$ are analytic functions of their arguments. We assume that the control (input) applied to system (1) is infinitely many times delta differentiable, i.e., $u^{[k]}$ exists for all $k \geq 0$. Let us define

$$\tilde{f}(x, u) := x + \mu f(x, u). \tag{2}$$