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Controllability of impulsive singularly perturbed systems and its application to a class of multiplex networks

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HIGHLIGHTS

- An effective analytical method is developed to solve the controllability problem of impulsive singularly perturbed systems.
- Both the sufficient and necessary controllable conditions are derived.
- The upper bound of singular perturbation parameter is deduced explicitly to ensure the controllability of the system.
- A new class of impulsive multilayer multi-time-scale networks is introduced and some controllability factors are revealed.

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ABSTRACT

This paper is concerned with the controllability problem of impulsive singularly perturbed systems (ISPSs). A new analytical approach is developed by integrating the merits of the fast–slow decomposition, Chang transformation and Gram-like matrix, and then some ε-independent necessary and sufficient controllable conditions are obtained. In addition, the upper bound of ε is given when deriving the sufficient controllable conditions. Moreover, a new type of heterogeneous multiplex multi-time-scale networks is introduced and can be further modeled by ISPSs. Based on matrix theory and graph theory, some intuitive and easy-to-test criteria are deduced for the controllability of the proposed networks. It is shown that the network topology, the nodal dynamics, the leader selection, and the inner-coupling interconnection are important controllable factors. Several numerical examples are presented to show the effectiveness of the proposed results.

1. Introduction

Singularly perturbed systems (SPSs) have received extensive attention and are widely used in past few decades due to their advantages in modeling many real systems that own both the fast and slow dynamics, such as mechanical systems [1], power systems [2], chemical reaction process [3], and biological networks [4]. In many real-word situations, due to instantaneous disturbances or abrupt changes during the evolution process of the systems at certain time instants, it is unavoidable that the abovementioned systems are subjected to impulsive effects [5,6]. Fortunately, impulsive singularly

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perturbed systems (ISPSs) provide a natural framework for characterizing them. Recently, great efforts have been devoted and many valuable results have been achieved for the analysis of ISPSs, please see [7–10] and the references therein.

As well known, the controllability property plays a crucial role in many control problems, such as feedback stabilization control and optimal control [11]. In past few decades, only a few results on the controllability of SPSs were obtained [12–18]. [12] studies the controllability property of linear time-invariant SPSs. Then, the established result in [12] is extended to linear time-varying SPSs [13], nonlinear SPSs [14], and multiparameter SPSs [15], respectively. Most recently, in [16–18] the Euclidean space controllability of SPSs with mixed time delays is investigated. Unfortunately, up to now, the controllability criterion for ISPSs has been overlooked in the existing literatures due to the difficulty in dealing with the discontinuity and multi-time-scale features of ISPSs. Also note that although the controllability issue of impulsive systems and its generality were made in [19–23], the proposed analysis methods and their results cannot be applied to deal with the controllability of ISPSs directly due to numerical ill-conditioning and stiffness problem resulted from singular perturbation parameter [24].

On the other hand, in recent years, controllability analysis has been extended from single system to multiple interaction networks [25–34]. Noticeably, the dynamic evolution of those models in these works are focused on the same time scale. However, in most realistic cases, the multi-time-scale properties are ubiquitous. For examples, in a typical transportation networks [35], there are different kinds of travel modes, such as airplane and car. In general, the former may take a few hour to arrive at the destination, while the latter needs about a few days to the same place. Another example is a company consisting of employees and managers, in which employees are in daily interaction whereas managers meet weekly [36]. Moreover, the evolution process of these examples may experience instantaneous disturbance or abrupt changes. For instance, the extreme weather conditions present a great challenge on the traveling; The government regulation plays an important role on the company's decision-making process. Common features of these examples are that (1) the whole networks are composed of a set of subnetworks that operate at different time scales; (2) impulsive effects have an important influence on the networks. Networks with these features can be referred to as impulsive multilayer multi-time-scale networks (IMMTSNs).

The above-mentioned discussion has motivated to address the following issues: (1) How can we develop an effective technique to establish controllability condition for ISPSs? (2) How can we properly build models for the IMMTSNs? (3) How do the networks parameters, such as network topology, nodal dynamics, the inner-coupling interconnection, and leader selection, affect the controllability of the proposed networks?

In this paper, the controllability issue of ISPSs is considered for the first time. Then, a new kind of IMMTSNs modeled by ISPSs is introduced. Moreover, the obtained controllable conditions for ISPSs are utilized to tackle the controllability of the presented networks. Based on matrix theory and graph theory, some intuitive and convenient controllable criteria are further deduced. The main contributions of this paper are listed as follows:

(1) An effective analytical approach, integrated use of the merits of the fast–slow decomposition, Chang transformation and Gram-like matrix, is developed to solve the controllability problem of ISPSs. Both the sufficient and necessary controllable conditions are obtained, which are new and encompass some existing results [12,24] as special cases. Moreover, the upper bound of ε is given when deriving the sufficient controllable conditions.

(2) A new class of IMMTSNs is introduced for modeling some realistic networks which are often the most neglected. Compared with the models in existing results [25–30], the most distinctive characteristics of the proposed networks lie in three aspects: a) all nodes are grouped into two layers that evolve at two different time scales; b) the nodal dynamics in different layers are heterogeneous; c) impulsive effects are also taken into the model.

(3) Some intuitive and easy-to-use criteria are presented for testing the controllability of the proposed networks. The obtained results show that the network topology, the nodal dynamics, the leader selection, and the inner-coupling interconnection are important factors affecting the controllability.

The rest of the paper is organized as follows. In Section 2, problem formulation and some preliminaries are given. In Section 3, some controllable criteria are deduced for ISPSs. In Section 4, a new kind of IMMTSNs is introduced firstly. Then, some controllable criteria are presented for the presented networks. In Section 5, some numerical examples are given to illustrate the validity of the proposed results. Finally, some conclusions and future works are drawn in Section 6.

Notations: $\mathbf{R}$ and $\mathbf{C}$ are respectively the real and complex numbers, $\mathbf{R}^n$ and $\mathbf{C}^n$ denote the vector space of real and complex n dimensional vectors, respectively; I_n is an n dimensional identity matrix. For a given matrix A, A^{T} refers to the matrix transposition, $\mathrm{rank}(A)$ and $\sigma(A)$ represent the rank and the spectrum of A, respectively; $\mathrm{diag}\{a_1, a_2, \ldots, a_n\}$ is the diagonal matrix with diagonal elements $a_1, a_2, \ldots, a_n$, $\otimes$ is the Kronecker product; $O(\varepsilon)$ denotes Landau order symbol.

2. Model description and preliminaries

In this section, model description and some preliminaries including definitions and lemmas are given.

Consider the following SPSs with impulsive effects

$$\begin{aligned}
&\begin{bmatrix} \dot{x}(t) \\ \varepsilon\dot{z}(t) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u(t), t \neq t_k, \\
&\begin{bmatrix} \Delta x(t_k) \\ \Delta z(t_k) \end{bmatrix} = \begin{bmatrix} d_k x(t_k^-) \\ h_k z(t_k^-) \end{bmatrix}, t = t_k, k = 1, 2, \ldots,
\end{aligned} \tag{1}$$