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Data fusion based multi-rate Kalman filtering with unknown input for on-line estimation of dynamic displacements

Ying Lei [a], Hao Qiu [a], Zhichao Wang [a], Sujuan Luo [b], Zhupeng Zheng [a,*]

[a] Department of Civil Engineering, Xiamen University, Xiamen 361005, China
[b] Fujian Institute of Technology, Fuzhou 350506, China

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ABSTRACT

Structural dynamic displacement is one of the most important measurands that describe the dynamic characteristics of a structure. However, accurate measurement of dynamic displacements of a civil infrastructure is still a challenging task. To solve the difficulties and drawbacks of direct dynamic displacement measurement, the approach of multi-rate Kalman filtering for the data fusion of displacement and acceleration measurement was developed. Recently, an improved technique for dynamic displacement estimation by fusing biased high-sampling rate acceleration and low-sampling rate displacement measurements has been proposed. However, this technique can only take constant acceleration bias into account. In this paper, based on the algorithm of Kalman filter with unknown input recently developed by the authors, dynamic displacement is on line estimated based on multi-rate data fusion of high-sampling rate acceleration with time-varying bias and low-sampling rate displacement measurements. The time history of time-varying acceleration bias is treated as "unknown input" in the algorithm of Kalman filter with unknown input to overcome the limitations of the previous technique. Some numerical examples with linear or polynomial acceleration bias are used to demonstrate the effectiveness of the proposed approach for on line estimation of structural dynamic displacement.

1. Introduction

The effective measurement of dynamic displacement is crucial not only to ensure the overall safety of engineering structures, but also to predict the abnormal state of structures [1–4]. Indeed, a variety of modern design codes adopt limit displacement levels under given loadings to assure structural safety [5]. Displacement, or deformation information is particularly important when nonlinear behavior and permanent deformations occur [6,7]. Measurement of dynamic displacement is also useful in structural control [8,9] and system identification applications [10,11]. In addition, displacement measurements have been used for bridge rating [12], seismic risk assessment [13], structural health monitoring of structures [14,15], etc.

However, displacement response measurement of existing structures is still difficult and cumbersome in practice. Since displacement is a relative physical quantity, it requires a reference. Thus, contact displacement sensors such as linear variable differential transformer (LVDT), which is one of the most common devices used to measure displacement in field, requires a direct contact of one end of LVDT with a target structure and the other end with a fixed scaffold or a firm support. This contact nature of LVDT makes its installation difficult, and the measurement by LVDT can be easily contaminated by support vibrations [16]. To resolve the limitation, some noncontact displacement sensors such as GPS, vision-based sensor, radar-based sensor and LiDAR (light detection and ranging) have been developed [17–23]. However, they still suffer from some problems such as high equipment cost, low sampling rate, and limited applicability.

There has been a strong preference of the use of measured acceleration to retrieve displacement, since accelerometers are commonly used in dynamic testing of structures due to their installation convenience, low cost and relatively high accuracy with low measurement noise [24]. The displacement estimation using acceleration measurements is based on the double integration of accelerations. However, the double integration involves intrinsic errors due to measurement noise and imperfect information in measured discrete acceleration signals, typically resulting in the low frequency drift in the estimated displacement [25]. Traditionally, this type of error is corrected by baseline correction or low and high pass filtering techniques [26,27]. Baseline correction is a least-square curve fitting technique in the time domain and filtering is a common noise removal technique in the frequency domain.

* Corresponding author.
E-mail address: xpeng@xmu.edu.cn (Z. Zheng).

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However, these techniques are not suitable for real-time estimation because the necessary post-processing can be performed only after the completion of data acquisition. There are ongoing efforts to address this non-unique nature of the displacement estimation by adopting displacement reconstruction techniques based on finite and infinite impulse filters [28,29] or frequency domain integration. However, all these techniques cannot estimate displacement properly when the mean value of the acceleration measurement is non-zero, or there are nonlinear or pseudo static components in displacement.

Alternatively, Kalman filters have been employed to obtain the optimal estimate of dynamic displacement by explicitly taking into account measurement errors when acceleration and intermittent displacement measurements are combined [1]. The estimation accuracy can be further improved by adopting fixed interval smoothing [24], but the employment of the fixed interval smoothing prevents real-time estimation of displacement because the smoothing techniques require more computational time and resources and hamper real-time estimation.

Recently, Kim et al. [30] presented a new dynamic displacement estimation technique based on two-stage Kalman estimator to further improve the accuracy of dynamic displacement. By adopting two-stage Kalman estimator, the proposed technique improves the convergence rate of Kalman gain and minimizes the level of discontinuities without Kalman filter smoothing. The performance of the proposed technique is verified by numerical simulations and experiments. However, it only considers acceleration bias with constant values. But some researchers have come up with that the measured acceleration has a linear or polynomial bias [31–33].

In this paper, dynamic displacement is estimated based on multi-rate Kalman filter with unknown input (KF-UI) using data fusion of partial acceleration and displacement measurements, which has recently been proposed by the authors [34]. In the proposed approach, the acceleration bias is regarded as an "unknown input" and multi-rate data fusion KF-UI algorithm is adopted to estimate the dynamic displacement. The proposed approach circumvented the above limitation, that is, when the acceleration bias is not a constant value, it also can estimate the dynamics displacement. Such algorithm is not available in the literature. Some numerical examples are used to demonstrate the effectiveness of the proposed approach.

The rest of the paper is organized as follows. In Section 2, the data fusion based Kalman filter with unknown input (KF-UI) algorithm recently developed by the authors [34] is briefly reviewed; In Section 3, the approach for real-time estimation of dynamic displacement using data fusion based multi-rate KF-UI is proposed. In Section 4, some numerical examples are used to validate the performances of the proposed approach. Finally, some concluding remarks and further necessary research works are presented in the conclusion section.

2. Brief review of the data fusion based KF-UI

When the external inputs to a linear structural system are unknown, the state equation of the system in the discrete form can be expressed as

$$\boldsymbol{X}_{k+1} = \boldsymbol{A}_k\boldsymbol{X}_k + \boldsymbol{G}_k\boldsymbol{f}_k^u + \boldsymbol{w}_k \tag{1}$$

where $\boldsymbol{X}_k$ is the state vector at time $t = k\Delta t$ with Δt being the sampling time step, $\boldsymbol{A}_k$ is the state transformation matrix, $\boldsymbol{f}_k^u$ denotes the unmeasured external input vector with the influence matrix $\boldsymbol{G}_k$, and $\boldsymbol{w}_k$ is the model noise (uncertainty) with zero mean and a covariance matrix $\boldsymbol{Q}_k$.

Analogous to the scheme of the classical Kalman Filtering (KF) approach, $\tilde{\boldsymbol{X}}_{k+1|k}$ is first predicted as,

$$\tilde{\boldsymbol{X}}_{k+1|k} = \mathbf{A}_k\hat{\boldsymbol{X}}_{k|k} + \boldsymbol{G}_k\hat{\boldsymbol{f}}^u_{k|k} \tag{2}$$

where $\tilde{\boldsymbol{X}}_{k+1|k}$ and $\hat{\boldsymbol{X}}_{k|k}$ denote the predicted $\boldsymbol{X}_{k+1}$ and estimated $\boldsymbol{X}_k$ at time $t = k\Delta t$, respectively. $\hat{\boldsymbol{f}}^u_{k|k}$ also denotes the estimated $\boldsymbol{f}^u$ at time $t = k\Delta t$.

In practice, only partial structural responses can be measured. The discrete form of the observation equation can be expressed as

$$\boldsymbol{Y}_{k+1} = \mathbf{C}_{k+1}\boldsymbol{X}_{k+1} + \boldsymbol{H}^u_{k+1}\boldsymbol{f}^u_{k+1} + \boldsymbol{v}_{k+1} \tag{3}$$

where $\boldsymbol{Y}_{k+1}$ is the measured response vector at time $t = (k+1)\Delta t$, $\mathbf{C}_{k+1}$ and $\boldsymbol{H}^u_{k+1}$ are measurement matrices with structural state and unmeasured input vector, respectively, and $\boldsymbol{v}_{k+1}$ is the measurement noise vector, which is assumed a Gaussian white noise vector with zero mean and a covariance matrix $\boldsymbol{R}_{k+1}$.

Then, the estimated $\boldsymbol{X}_{k+1}$ in the measurement update (correction) procedure is derived as

$$\hat{\boldsymbol{X}}_{k+1|k+1} = \tilde{\boldsymbol{X}}_{k+1|k} + \boldsymbol{K}_{k+1}(\boldsymbol{Y}_{k+1} - \boldsymbol{C}_{k+1}\tilde{\boldsymbol{X}}_{k+1|k} - \boldsymbol{H}^u_{k+1}\hat{\boldsymbol{f}}^u_{k+1|k+1}) \tag{4}$$

where $\boldsymbol{K}_{k+1}$ is the Kalman gain matrix given by

$$\boldsymbol{K}_{k+1} = \tilde{\mathbf{P}}^{\boldsymbol{X}}_{k+1|k}\boldsymbol{C}^{\mathrm{T}}_{k+1}\left(\boldsymbol{C}_{k+1}\tilde{\mathbf{P}}^{\boldsymbol{X}}_{k+1|k}\boldsymbol{C}^{\mathrm{T}}_{k+1} + \boldsymbol{R}_{k+1}\right)^{-1} \tag{5}$$

in which $\tilde{\boldsymbol{P}}^{\boldsymbol{X}}_{k+1|k}$ denotes the error covariance matrices of the predicted state vector X by Eq. (2). The subscript $k+1|k$ denotes the estimation value at time step $k+1$ with the observation at time step k.

Under the condition that the number of measurements (sensors) is no less than that of the unknown inputs, $\hat{\boldsymbol{f}}^u_{k+1|k+1}$ can be estimated by minimizing the error vector Δ_{k+1} defined by:

$$\begin{aligned}\Delta_{k+1} &= \boldsymbol{Y}_{k+1} - \boldsymbol{C}_{k+1}\hat{\boldsymbol{X}}_{k+1|k+1} - \boldsymbol{H}^u_{k+1}\hat{\boldsymbol{f}}^u_{k+1|k+1} \\ &= (\boldsymbol{I} - \boldsymbol{C}_{k+1}\boldsymbol{K}_{k+1})(\boldsymbol{Y}_{k+1} - \boldsymbol{C}_{k+1}\tilde{\boldsymbol{X}}_{k+1|k}) \\ &\quad -(\boldsymbol{I} - \boldsymbol{C}_{k+1}\boldsymbol{K}_{k+1})\boldsymbol{H}^u_{k+1}\hat{\boldsymbol{f}}^u_{k+1|k+1}\end{aligned} \tag{6}$$

Then, $\hat{\boldsymbol{f}}^u_{k+1|k+1}$ can be estimated from Eq. (6) based on the least-squares estimation as

$$\hat{\boldsymbol{f}}^u_{k+1|k+1} = \boldsymbol{S}_{k+1}\boldsymbol{H}^{uT}_{k+1}\boldsymbol{R}^{-1}_{k+1}(\boldsymbol{I} - \boldsymbol{C}_{k+1}\boldsymbol{K}_{k+1})\left[\boldsymbol{Y}_{k+1} - \boldsymbol{C}_{k+1}\tilde{\boldsymbol{X}}_{k+1|k}\right] \tag{7}$$

in which,

$$\boldsymbol{S}_{k+1} = \left[\boldsymbol{H}^{uT}_{k+1}\boldsymbol{R}^{-1}_{k+1}(\boldsymbol{I} - \boldsymbol{C}_{k+1}\boldsymbol{K}_{k+1})\boldsymbol{H}^u_{k+1}\right]^{-1} \tag{8}$$

Also, the error covariance matrices can be derived as [34]

$$\begin{aligned}\hat{\boldsymbol{P}}^{\boldsymbol{X}}_{k+1|k+1} &= E\left[\hat{\boldsymbol{e}}^{\boldsymbol{X}}_{k+1|k+1}\ \ (\hat{\boldsymbol{e}}^{\boldsymbol{X}}_{k+1|k+1})^T\right] \\ &= (\boldsymbol{I} + \boldsymbol{K}_{k+1}\boldsymbol{D}_{k+1}\boldsymbol{S}_{k+1}\boldsymbol{D}^{\mathrm{T}}_{k+1}\boldsymbol{R}^{-1}_{k+1}\boldsymbol{C}_{k+1})(\boldsymbol{I} - \boldsymbol{K}_{k+1}\boldsymbol{C}_{k+1})\tilde{\boldsymbol{P}}^{\boldsymbol{X}}_{k+1|k}\end{aligned} \tag{9}$$

$$\begin{aligned}\hat{\boldsymbol{P}}^{\boldsymbol{f}}_{k+1|k+1} &= E\left[\hat{\boldsymbol{e}}^{\boldsymbol{f}}_{k+1|k+1}\ \ (\hat{\boldsymbol{e}}^{\boldsymbol{f}}_{k+1|k+1})^T\right] \\ &= \boldsymbol{S}_{k+1}\boldsymbol{D}^{\mathrm{T}}_{k+1}\boldsymbol{R}^{-1}_{k+1}(\boldsymbol{I} - \boldsymbol{C}_{k+1}\boldsymbol{K}_{k+1}) \\ &\quad \times(\boldsymbol{C}_{k+1}\boldsymbol{P}^{\sim\boldsymbol{X}}_{k+1|k}\boldsymbol{C}^T_{k+1} + \boldsymbol{R}_{k+1}) \\ &\quad \times(\boldsymbol{I} - \boldsymbol{C}_{k+1}\boldsymbol{K}_{k+1})^T\boldsymbol{R}^{-T}_{k+1}\boldsymbol{D}_{k+1}\boldsymbol{S}^{\mathrm{T}}_{k+1} \\ &= \boldsymbol{S}_{k+1}\boldsymbol{D}^{\mathrm{T}}_{k+1}(\boldsymbol{I} - \boldsymbol{C}_{k+1}\boldsymbol{K}_{k+1})^T\boldsymbol{R}^{-T}_{k+1}\boldsymbol{D}_{k+1}\boldsymbol{S}^{\mathrm{T}}_{k+1} = \boldsymbol{S}_{k+1}\end{aligned} \tag{10}$$

$$\hat{\boldsymbol{P}}^{\boldsymbol{Xf}}_{k+1|k+1} = E\left[\hat{\boldsymbol{e}}^{\boldsymbol{X}}_{k+1|k+1}\ \ (\hat{\boldsymbol{e}}^{\boldsymbol{f}}_{k+1|k+1})^T\right] = (\hat{\boldsymbol{P}}^{\boldsymbol{Xf}}_{k+1|k+1})^T = -\boldsymbol{K}_{k+1}\boldsymbol{H}^u_{k+1}\boldsymbol{S}_{k+1} \tag{11}$$

$$\tilde{\boldsymbol{P}}^{\boldsymbol{X}}_{k+1|k} = E\left[\hat{\boldsymbol{e}}^{\boldsymbol{X}}_{k+1|k}\ \ (\hat{\boldsymbol{e}}^{\boldsymbol{X}}_{k+1|k})^T\right] = [\mathbf{A}_k\ \ \boldsymbol{G}_k]\begin{bmatrix}\hat{\boldsymbol{P}}^X_{k|k} & \hat{\boldsymbol{P}}^{Xf}_{k|k} \\ \hat{\boldsymbol{P}}^{fX}_{k|k} & \hat{\boldsymbol{P}}^{f}_{k|k}\end{bmatrix}\begin{bmatrix}\boldsymbol{A}^T_k \\ \boldsymbol{G}^T_k\end{bmatrix} + \boldsymbol{Q}_k \tag{12}$$