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Mixed-dimensional coupling method for box section member based on the optimal stress distribution pattern

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A B S T R A C T

Mixed-dimensional coupling is significant for the accurate performance analysis of a concurrent multi-scale model, while the stress distribution pattern is a key problem that influences the mixed-dimensional coupling method. With respect to existing coupling methods, the real stress pattern of the complicated section is replaced by the approximate formula, the uniform distribution pattern of shear stress is used along the sectional thickness direction, and the shear stress of the sectional corner is neglected. The aforementioned simplified assumptions of stress distribution pattern are the key reasons that cause a distortion of the local stress and reduce the reliability of the multi-scale model. In the study, the optimal stress distribution pattern is determined using the proposed method, in order to provide the optimal stress distribution for mixed-dimensional coupling, and improve the accuracy of the multi-scale model. First, a stress distribution calculation model based on the higher-dimensional element type and section size of member is established. Next, the stress distribution pattern is determined and optimized using the surface fitting technology and iterative calculation, in which the coefficient of determination R^2 is selected as the optimization index. Third, under the specified local coordinate system, the multipoint constraint equations for mixed-dimensional coupling are determined based on the optimal stress distribution patterns and element shape functions. Finally, the optimal stress distribution patterns of box section, which consider the linear distribution along the thickness direction and stress contribution in the sectional corner, are determined. Furthermore, concurrent multi-scale models of a cantilever beam and warren truss structure with the box section are established, and are used to validate the effectiveness and applicability of the proposed method based on a comparison of the structural global responses, local responses, and dynamic characteristics.

1. Introduction

For the large civil engineering structures, loads act on the macro-scale of the structure, while the damage occurs on the local micro-scale of the structure [1]. When the lower-dimensional elements are used to establish the structural global simplified model, the finite element (FE) model cannot reflect the structural local mechanical properties and failure mechanism, due to the lack of the description and simulation of the local details. That is to say, the FE model based on the lower-dimensional elements can only be used to study the global mechanical properties of the structure. On the other hand, the full-scale refined FE model based on the higher-dimensional elements, not only can understand the structural global performance, but also accurately simulate the mechanical properties and damage characteristics of the local details of the structure. However, when the sizes of the civil structure are large, there will be excessive number of nodes and elements in the full-scale refined FE model [2,3], which will lead to complexity and time-consuming for modelling and calculation. Moreover, too much degrees of freedom of the full-scale refined model may cause data overflow and computational stop, while current computing hardware and software conditions are still difficult to solve the requirements of computational scale and storage space. Concurrent multi-scale model adopts a modelling strategy: the lower-dimensional element is used to realize the simplified simulation of the whole structure; simultaneously, the higher-dimensional element is used to realize the refined simulation of the structural key parts of interest, in order to avoid the disadvantages of the structural global simplified model and full-scale refined model. Thus concurrent multi-scale model is a balance solution between accuracy and computational cost [4].

Concurrent multi-scale modelling simultaneously analyses a model based on different dimensional elements by the coupling

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technique [5]. This type of model reflects the structural different scale characteristics and provides a thorough understanding of the behaviour of the entire structure and key parts. This is an increasing concern in the field of civil engineering [6–8]. The key to establishing a concurrent multi-scale model is mixed-dimensional coupling that primarily includes transition element method, the Arlequin method, and the multipoint constraint (MPC) equation method. The transition element method is only used to create a one-to-one coupling of elements [9], and different formulations are required for different element transitions [10,11]. Thus this is difficult and impractical for building a finite element model of a large complex civil structure using the commercial FE code. The Arlequin method requires that different models have overlapping areas, while the length of the overlapping area is artificially set, which causes that this coupling method exhibits strong subjectivity [12]. In addition, the determination of energy distribution of each model in the overlapping area is a necessary step for using the coupling operator to realise the coupling [13,14], and thus the modelling process based on this method is tedious which limits the application of the Arlequin method. The MPC equation method realises mixed-dimensional coupling by establishing constraint equations between the degrees of freedom of nodes on both sides of the connected interface. The methods to establish MPC equations include a method based on the plane section assumption and deformation compatibility [15,16], and a method based on the energy conservation [17–20]. With respect to the former method, the internal association relation between the degrees of freedom of nodes can be directly determined, however, the plane section assumption is not tenable in terms of the shear force. This method reduces the calculation accuracy of the concurrent multi-scale model near the connected interface. With respect to the latter method, the calculation accuracy of the multi-scale model based on energy conservation is only as good as the accuracy of the assumed stress distribution, and thus the stress distribution pattern at the connected interface is a crucial step for establishing the concurrent multi-scale model.

The MPC equation method based on the energy conservation was developed by McCune [21] and Monaghan [22], where the mixed-dimensional coupling was achieved using the Reissner's bending theory [23] and the assumed stress distributions. Subsequently, the multi-scale model of a cantilever member with a reduced beam section was established using this coupling method, and Hailu et al. [24] emphasised that constraint equations were derived based on an assumed stress distribution at the interface, thus the accuracy of the constraint equations is closely related to the assumed stress distribution. However, Zheng et al. [25]. pointed out that the stress distribution at the connected interface, which was essential for mixed-dimensional coupling, cannot be solved for arbitrary types of cross section, especially for the distribution of shear stress. That is to say, the limited stress formulae in material mechanics limit the type of cross section of the structural multi-scale model. For example, Yue et al. [26] used the stress formula of a thin-walled circular section to establish a multi-scale model of the pipe column. Yan et al. [27] used the stress formula of a rectangular section to establish a multi-scale model of the simply supported beam. According to the suggestion of Gere and Timoshenko [28], Sadeghian et al. [29] estimated the shear strain distribution of the rectangular section and circular section to model beam-membrane interface. In the above studies, the introduction of the stress formula limits its application to only the rectangular and circular section. In fact, because of the good bending and torsion resistance, light weight, and low consumables, the box section has been widely used in bridges and large span space steel structures in recent years [30–33]. Obviously, these shear stress formulae are not suitable for the box section. For this problem, Yu et al. [20] established a multi-scale model of steel truss with box section based on the simplified stress formulae in material mechanics, where the shear stress formula of the closed thin-walled section under the torque and the I-shaped section under the shear force were used. With respect to the simplified formulae, the shear stress along the thickness of section is constant, while the distribution of shear stress along the thickness does not always correspond to a constant. In addition, when the simplified formulae in material mechanics are used to represent the real shear stress distribution pattern of the box section, the simplified shear stress distribution based on the approximate cross section rather rough. The result is that the calculation accuracy of a multi-scale model near the connected interface is reduced and the local stress concentration is obvious. Therefore, it is necessary to optimize the stress distribution pattern at the connected interface to improve the accuracy of the concurrent multi-scale model with box section.

In this study, a mixed-dimensional coupling method for box section member is proposed to provide the optimal stress distribution patterns, improve the calculation accuracy of the concurrent multi-scale model, and obtain the accurate structural responses. First, the determination method of the optimal stress distribution pattern is elaborated in detail, and this includes the establishment of stress distribution calculation model, optimization and evaluation of stress distribution pattern. Second, the MPC equations based on the optimal stress distribution patterns and element shape functions are derived under a specified local coordinate system. Third, the selection strategy of the element shape functions is described. Finally, the optimal stress distribution patterns of the box section are determined using the proposed method. The concurrent multi-scale models of a cantilever member and warren truss with box section are established to validate the effectiveness and applicability of the proposed method.

2. Determination of the optimal stress distribution pattern at the connected interface

2.1. Establishment of stress distribution calculation model

Stress distribution calculation model is the basis for determining the optimal stress distribution pattern, which is equivalent to a microcosm of the higher-dimensional part of the concurrent multi-scale model, and thus they include the same element type, shape function and the cross-section size. In addition, the following two main points should be considered: (1) Based on Saint-Venant's principle, the length of the calculation model is 3–5 times the length of the long side of the box section, in order to avoid the cumbersome calculations and boundary effect on the stress distribution. (2) Considering the stress distribution of the cross section is affected by mesh size, the initial mesh size along the thickness direction of the calculation model is set as $t/2$.

A stress distribution calculation model is established as shown in Fig. 1. It consists of a main body based on the higher-dimensional element type, constraint point and loading point at the centroid of the end of the main body, and the rigid connection between the loading points and main body. The coordinate system of the calculation model is set such that the coordinate origin is located at the centroid line of the cross section; the x axis (1 axis) is parallel to the axis direction of the calculation model; the y axis (2 axis) and z axis (3 axis) are parallel to the bidirectional symmetry axes of the cross section.

2.2. Optimization and evaluation of stress distribution pattern

Based on the stress distribution calculation model, the fixed end constraint is applied to the constraint point, and the unit concentrated force and moment are applied to the loading point, respec-